



The Mathematical Association of Victoria FURTHER MATHEMATICS

Trial written examination 1 (Facts, skills and applications)

2007

Reading time: 15 minutes

Writing time: 1 hour 30 minutes

Student's Name:

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	13	13	6	3	13
B	54	27			27
					Total 40

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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MULTIPLE CHOICE ANSWER SHEET

Student Name:

Circle the letter that corresponds to each correct answer

Section A	Section B		
Compulsory	Answer three different modules. Show each module selected by ticking the appropriate box.		
	Module:	Module:	Module:
	☐ Number patterns ☐ Geometry and trigonometry ☐ Graphs and relations ☐ Business related mathematics ☐ Networks and decision mathematics ☐ Matrices	☐ Number patterns ☐ Geometry and trigonometry ☐ Graphs and relations ☐ Business related mathematics ☐ Networks and decision mathematics ☐ Matrices	☐ Number patterns ☐ Geometry and trigonometry ☐ Graphs and relations ☐ Business related mathematics ☐ Networks and decision mathematics ☐ Matrices
1. A B C D E	1. A B C D E	1. A B C D E	1. A B C D E
2. A B C D E	2. A B C D E	2. A B C D E	2. A B C D E
3. A B C D E	3. A B C D E	3. A B C D E	3. A B C D E
4. A B C D E	4. A B C D E	4. A B C D E	4. A B C D E
5. A B C D E	5. A B C D E	5. A B C D E	5. A B C D E
6. A B C D E	6. A B C D E	6. A B C D E	6. A B C D E
7. A B C D E	7. A B C D E	7. A B C D E	7. A B C D E
8. A B C D E	8. A B C D E	8. A B C D E	8. A B C D E
9. A B C D E	9. A B C D E	9. A B C D E	9. A B C D E
10. A B C D E			
11. A B C D E			
12. A B C D E			
13. A B C D E			

Working space

CORE: DATA ANALYSIS

Instructions for Multiple-choice questions

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

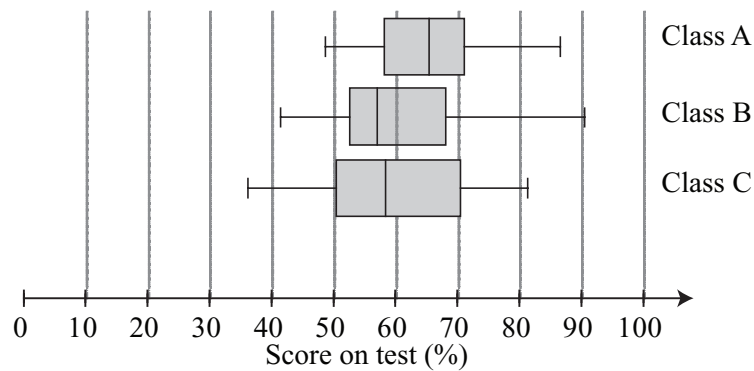
Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Multiple-choice questions (13 questions)

The following information relates to questions 1, 2 and 3

The graph below shows the results (percentages) for three different classes on a recent test.



Question 1

The variables **Class** and **Score on test** are respectively

- A. both categorical variables
- B. both numerical variables
- C. a categorical and a numerical variable
- D. a numerical and a categorical variable
- E. neither a categorical nor numerical variable

Question 2

The interquartile range for Class C is approximately

- A. from 58% to 81%
- B. from 36% to 81%
- C. 20%
- D. 45%
- E. 58%

Question 3

Which one of the following is **not** true for the data

- A. Scores in class A were generally higher
- B. The highest mark but the lowest median were in class B
- C. More than 75% of the students obtained a score greater than 50%
- D. The distribution of scores in class B was negatively skewed
- E. 75% of the students in class A obtained a score that was greater than the median for classes B and C

Question 4

The weights of 2kg bags of onions are normally distributed with a mean weight of 2 kg and a standard deviation of 0.05 kg.

The percentage of bags of onions that have a weight between 2.05kg and 2.10 kg is

- A. 2.5%
- B. 5%
- C. 13.5%
- D. 16%
- E. 34%

Question 5

Maria scored 56% on her recent Physics test where the scores were normally distributed around a mean mark of 66%. Maria was told that her z-score was -1.25 for this test.

The standard deviation for this Physics test was

- A. 5%
- B. 8%
- C. 10%
- D. 12%
- E. 12.5%

The following information relates to questions 6 and 7

A two-way frequency table has been used to display the results of a survey of 120 people on their voting intention at the next election. The people surveyed have been grouped into three age groups to see if age is a factor in their voting intention.

		<i>Age group</i>			
		18 - < 30	30 - < 50	50 ⁺	Total
<i>Voting intention</i>	Same party	20	18	23	61
	Different party	9	15	18	42
	Undecided	4	9	4	17
	Total	33	42	45	120

Question 6

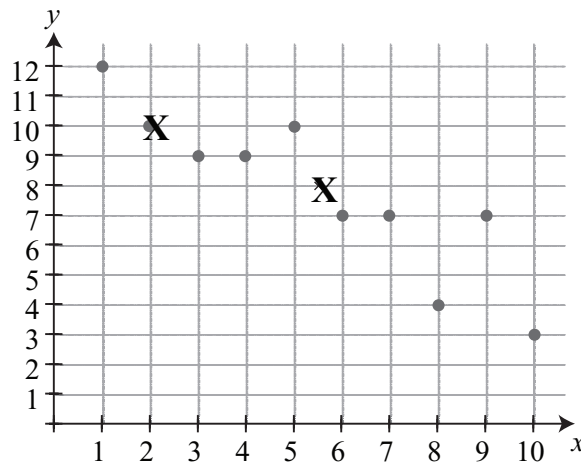
The percentage of those surveyed in the 50⁺ age group that are going to vote for the same party as previously is

- A. 19.2%
- B. 23%
- C. 37.5%
- D. 37.7%
- E. 51.1%

Question 7

Which one of the following statements is not a true statement based on the results of the survey?

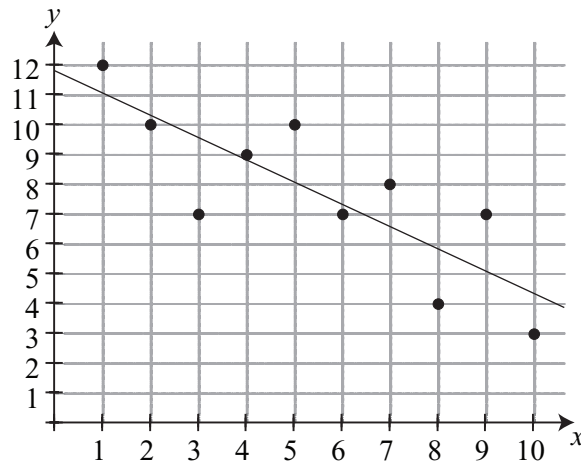
- A. More than half of the people surveyed intended to vote for the same party.
- B. 35% of the people surveyed intended to vote for a different party.
- C. A larger percentage of those in the 50^+ age group compared to those in the other two age groups intended to vote for a different party.
- D. The greatest percentage of undecided voters was in the $30 - < 50$ age group.
- E. A larger percentage of the 50^+ age group compared to the other two age groups are going to vote for the same party.

Question 8

The left and middle median points required to construct a three-median regression line have been located, and are marked X, on the scatterplot above. The right median point is not marked.

The gradient of the three-median regression line, correct to one decimal place, will be

- A. -1.2
- B. -1.1
- C. -1.0
- D. -0.9
- E. -0.7

Question 9

A least squares regression line has been fitted to the scatterplot above.

Which one of the following statements is **true** for the scatterplot and the regression line?

- A. The regression line overestimates the value for $x = 5$.
- B. There are more negative residuals than positive residuals.
- C. The largest residual is approximately +2.5.
- D. The gradient of the regression line is less than -1.
- E. The value predicted using the regression line for the point (1, 12) is the point (1, 11).

Question 10

4-point moving-mean smoothing, with centering, is being used to smooth the following time-series data.

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Incidents	38	44	29	26	25	22	29	32	40	35	34	48

The smoothed number of incidents for the month of May will be

- A. 25
- B. 25.5
- C. 26.5
- D. 29
- E. 32.5

The following information relates to questions 11 and 12

The following table shows the quarterly sales figures (in \$1000's) for 2005 and 2006 as well as the yearly averages for these two years. The seasonal indexes, correct to four decimal places have been calculated for Quarters 1 and 2.

	Quarter				
Year	1	2	3	4	Yearly average
2005	190	254	279	173	224
2006	194	266	283	181	231
Seasonal Index	0.8440	1.1427			

Question 11

The seasonal index for Quarter 3, correct to four decimal places, will be

- A. 1.0103
- B. 1.2264
- C. 1.2351
- D. 1.2353
- E. 1.2432

Question 12

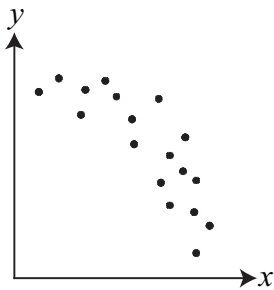
When the data is deseasonalised the sales figures for Quarters 1 and 2 of 2005 will

- A. increase and decrease respectively
- B. decrease and increase respectively
- C. both increase
- D. both decrease
- E. remain the same

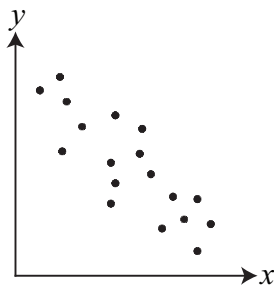
Question 13

Which one of the following scatterplots could be linearised using a $\frac{1}{y}$ transformation?

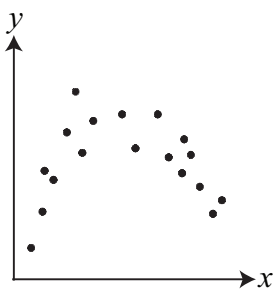
A.



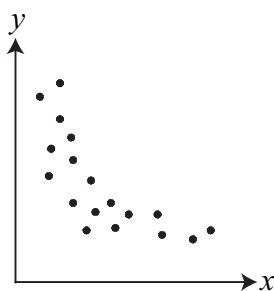
B.



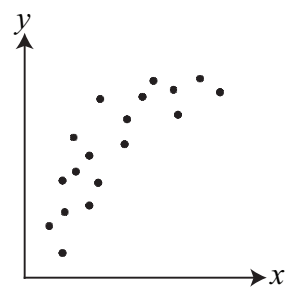
C.



D.



E.



MODULE 1: NUMBER PATTERNS**Instructions**

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions.

Question 1

The sum of the first twelve terms of the sequence 17, 23, 29, 35, . . . is

- A. 77
- B. 83
- C. 564
- D. 600
- E. 1620

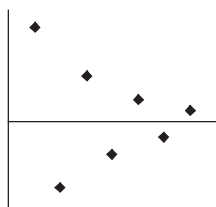
Question 2

A geometric sequence has a common ratio of 1.2 and the third term has the value 180.
The first term to exceed 1000 will be term

- A. 12
- B. 13
- C. 14
- D. 17
- E. 18

Question 3

The first seven terms of two sequences are plotted on the graphs shown



Sequence I



Sequence II

Which of the following statements is true?

- A. Both sequences are arithmetic
- B. Both sequences are geometric
- C. Sequence I is arithmetic and Sequence II is geometric
- D. Sequence I is geometric and Sequence II is arithmetic
- E. Neither sequence is arithmetic or geometric

Question 4

Adam loses 8 kg in the first month of exercise, 4.8 kg in the second month, 2.88kg in the third month and so on. If this continues, the total weight loss in kg will be

- A. 13.3
- B. 16.6
- C. 19.9
- D. 20.0
- E. 23.3

Question 5

A wildlife service is monitoring the number of koalas near a suburban area for a period of n years. During this time it has found that the number of koalas is decreasing by 6% each year. There were 2500 koalas when they were first counted.

Let K_n be the number of koalas in the area n years after counting. A difference equation describing this situation is

- A. $K_n = -0.06 \times K_{n-1}$ where $K_0 = 2500$
- B. $K_n = K_{n-1} - 0.06$ where $K_0 = 2500$
- C. $K_n = 1.06 \times K_{n-1}$ where $K_0 = 2500$
- D. $K_n = K_{n-1} + 0.06$ where $K_0 = 2500$
- E. $K_n = 0.94 \times K_{n-1}$ where $K_0 = 2500$

Question 6

To stop the decline in the koala population the wildlife service decides to introduce 200 koalas into a wildlife reserve. The number of koalas in the wildlife reserve at the beginning of the n th year is given by the difference equation $W_n = 0.88 \times W_{n-1} + 50$ where $W_1 = 200$

The koala population in this wildlife reserve will first exceed 400 at the start of

- A. the 20th year
- B. the 21st year
- C. the 22nd year
- D. the 23rd year
- E. the 24th year

Question 7

A sequence is defined by the difference equation $t_{n+1} = -4t_n - 1$. If the third term of the sequence is 51, the first term is

- A. -205
- B. -13
- C. 13
- D. 5
- E. 3

Question 8

A second order difference equation is defined by $T_{n+2} = 2T_n + T_{n+1}$, where $T_1 = 2$ and $T_2 = 5$.
The value of T_8 is

- A. 299
- B. 296
- C. 292
- D. 288
- E. 282

Question 9

In the Fibonacci sequence, $f_{12} = 555$ and $f_{10} = 212$
The value of the 11th term is

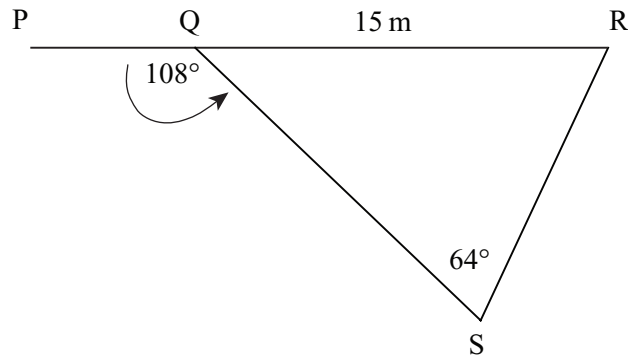
- A. 171.5
- B. 343
- C. 383.5
- D. 424
- E. 767

MODULE 2: GEOMETRY AND TRIGONOMETRY

Instructions

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

The following information relates to Questions 1 and 2



Question 1

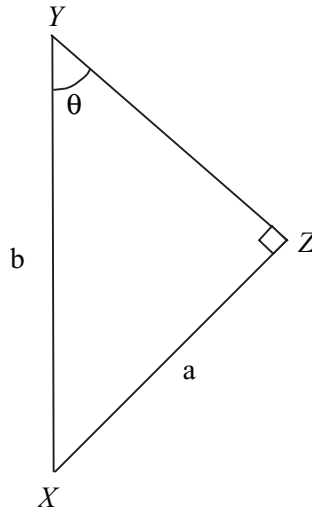
The size of $\angle QRS$ is exactly

- A. 44°
- B. 72°
- C. 64°
- D. 79°
- E. 136°

Question 2

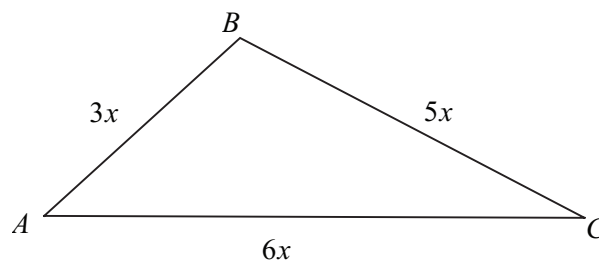
Given that the length of QR is 15 m, the length of RS is

- A. $15 \tan 64^\circ$
- B. $\frac{15}{\tan 64^\circ}$
- C. $\frac{15 \sin 44^\circ}{\sin 64^\circ}$
- D. $\frac{15 \sin 64^\circ}{\sin 72^\circ}$
- E. $\frac{15 \sin 72^\circ}{\sin 64^\circ}$

Question 3

In the right-angled $\triangle XYZ$, the length of $XZ = a$, the length of $XY = b$ and $\angle XYZ = \theta$.
For the triangle, $\tan \theta$ is given by

- A. $\frac{a}{b}$
- B. $\frac{a}{\sqrt{b^2 + a^2}}$
- C. $\frac{\sqrt{b^2 - a^2}}{a}$
- D. $\frac{b}{\sqrt{b^2 - a^2}}$
- E. $\frac{a}{\sqrt{b^2 - a^2}}$

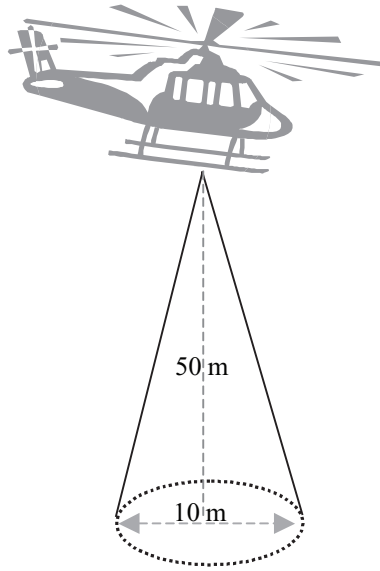
Question 4

In the $\triangle ABC$, the length of $AB = 3x$, the length of $BC = 5x$ and the length of $AC = 6x$.
The area of $\triangle ABC$ is given by

- A. $9x^2$
- B. $15x^2$
- C. $\sqrt{14}x^2$
- D. $\sqrt{15}x^2$
- E. $\sqrt{56}x^2$

The following information relates to Questions 5 and 6

A search and rescue helicopter shines a light down from a vertical height of 50 metres as shown below. The circular area of light it creates on the ground has a diameter of 10 metres.



Question 5

The helicopter is elevated an additional 15 metres away from the ground. The diameter of the circular area of light on the ground is now closest to

- A. 8 m
- B. 13 m
- C. 15 m
- D. 20 m
- E. 25 m

Question 6

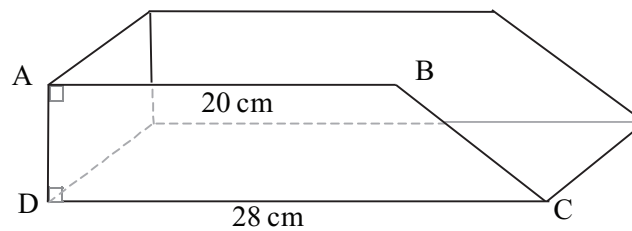
The helicopter moves to a height so that the diameter of the circular area increases from 10 metres to 40 metres.

The area of the circular light is now

- A. four times what it was before.
- B. eight times what it was before.
- C. sixteen times what it was before.
- D. thirty-two times what it was before.
- E. sixty-four times what it was before.

Question 7

An open paper tray is made of a rectangular base and three sides as shown.



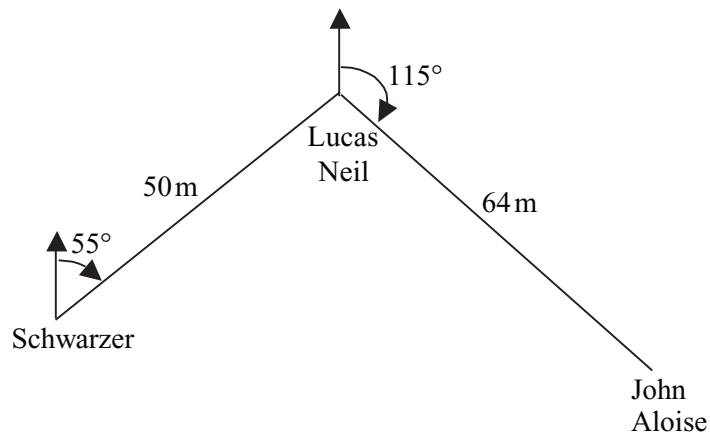
The area of side ABCD is 240 square centimetres.

The height of the tray AD is closest to

- A. 5 cm
- B. 8 cm
- C. 9 cm
- D. 10 cm
- E. 12 cm

Question 8

Goal keeper Schwarzer kicks a soccer ball to Lucas Neil 50 metres away at a bearing of 055°T . Lucas Neil then kicks the ball to John Aloise 64 metres away on a bearing of 115°T .



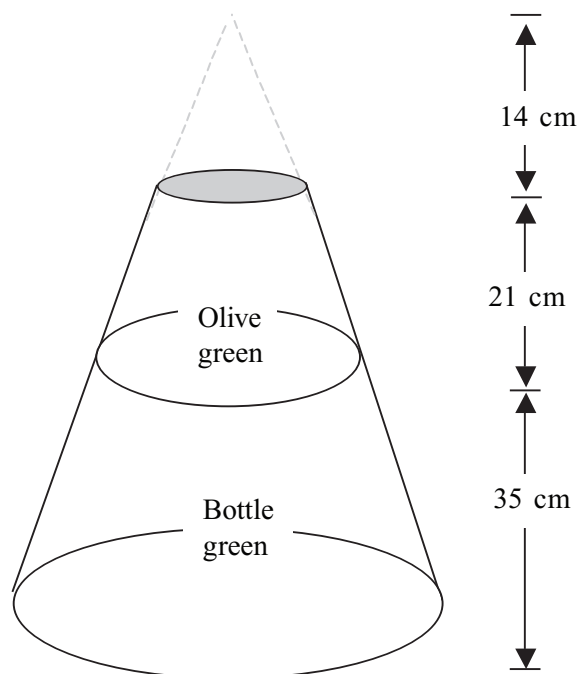
The distance between John Aloise and Schwarzer is closest to

- A. 58 m
- B. 62 m
- C. 73 m
- D. 81 m
- E. 99 m

Question 9

A skirt is to be made of two shades of green material, olive green along the top section and bottle green along the bottom section.

The shape of the skirt is a truncated cone as shown



The ratio representing the area of olive green material to the area of bottle green material is

- A. 7: 25
- B. 3: 5
- C. 9: 25
- D. 1: 2
- E. 1: 4

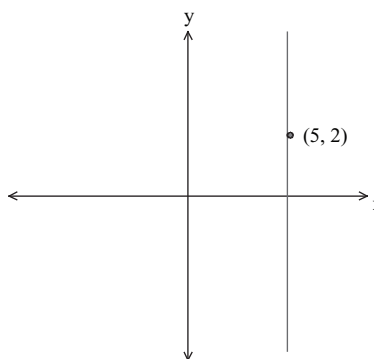
MODULE 3: GRAPHS AND RELATIONS**Instructions**

Before answering these questions you must **shade** the Graphs and Relations box on the answer sheet for multiple-choice questions.

Question 1

The graph shown has equation

- A. $y = 5$
- B. $y = 2$
- C. $x = 5$
- D. $x = 2$
- E. $5x + 2y = 0$

**Question 2**

For the straight line with equation $3x + 5y = 30$, which of the following statements is false?

- A. as x decreases, y increases
- B. the y -intercept is 6
- C. the x -intercept is 10
- D. it has a gradient of $-\frac{5}{3}$
- E. the line passes through the point (5, 3)

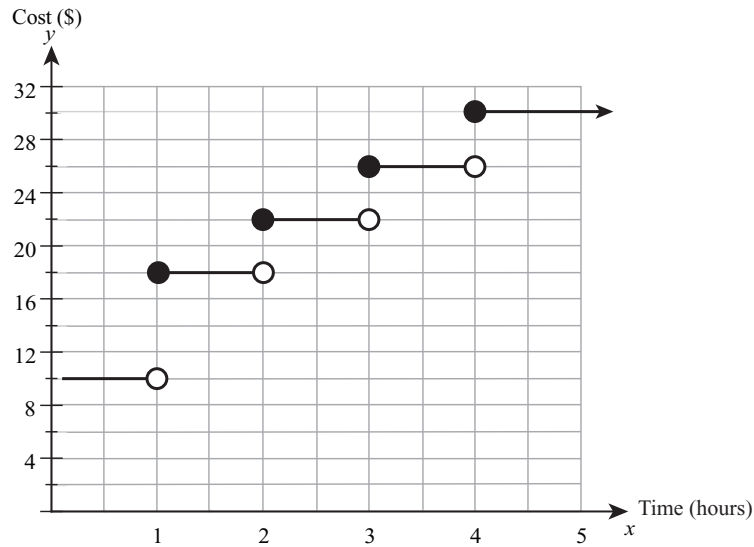
Question 3

At the craftmarket, Helga sold 3 scarves and 2 hats for \$60. She later sold 5 hats and 2 scarves for \$73. From this information, one hat costs

- A. \$6.50
- B. \$9.00
- C. \$14.00
- D. \$16.50
- E. \$20.25

Question 4

The graph below shows the cost of parking in a Melbourne city carpark.



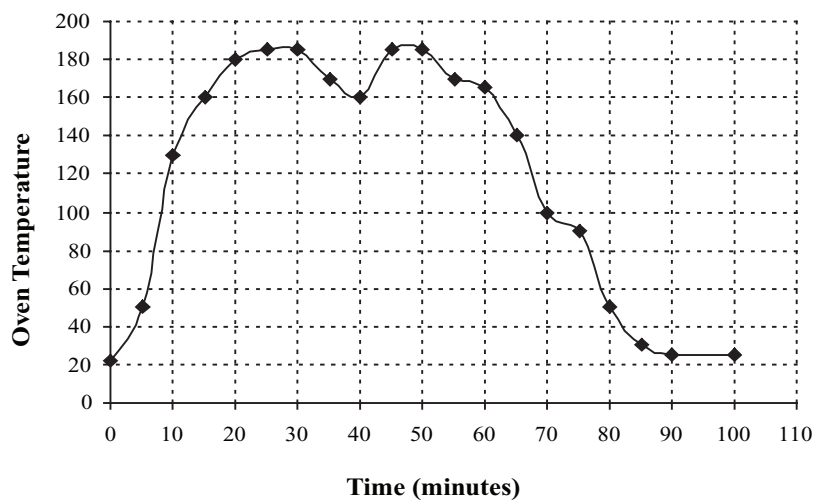
John and Betty parked their car in the same carpark. John paid $\$x$ and Betty paid $\$y$. The sum of their parking costs $x + y = \$48$.

If John parked his car for 2 hours then Betty parked her car for

- A. less than one hour
- B. one hour or more but less than two hours
- C. two hours or more but less than three hours
- D. three hours or more but less than four hours
- E. more than four hours

Question 5

The graph below shows the temperature change (in degrees Celsius) in an oven over a period of 100 minutes.

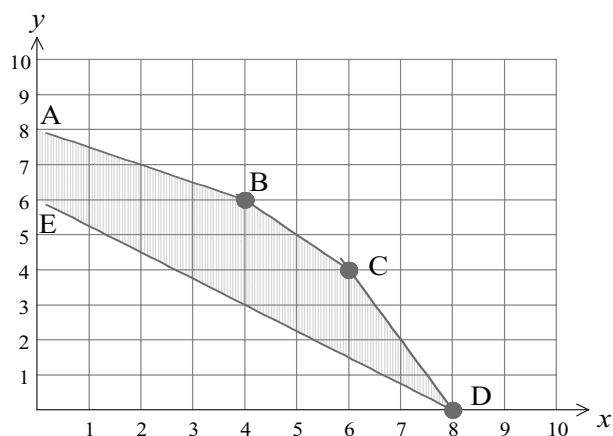


The period of greatest temperature decrease was

- A. 0 – 20 minutes
- B. 20 – 40 minutes
- C. 40 – 60 minutes
- D. 60 – 80 minutes
- E. 80 – 100 minutes

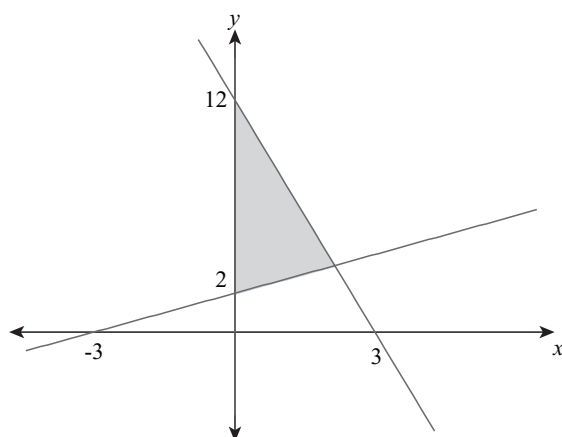
Question 6

The feasible region for a particular linear programming problem is shaded on the following graph. All the relevant vertices are labelled.



The maximum value of the expression $2x + 4y$ occurs at

- A. point A
- B. point B
- C. point A and B
- D. the points on the line joining points A and B
- E. the points on the line joining points B and C

Question 7

The shaded region shown in the graph above (with boundaries included) is described by

- | | |
|---------------------|---------------------|
| A. $y + 4x \leq 12$ | B. $y - 4x \leq 12$ |
| $3y - 2x \geq 6$ | $3y - 2x \geq 6$ |
| $x \geq 0$ | $x \geq 0$ |
| C. $y + 4x \geq 12$ | D. $y - 4x \leq 12$ |
| $3y - 2x \geq 6$ | $3y - 2x \leq 6$ |
| $x \geq 0$ | $x \geq 0$ |
| E. $y + 4x \leq 12$ | |
| $3y - 2x \geq 6$ | |
| $x \leq 0$ | |

Question 8

The local supermarket uses permanent staff and casual staff.

The permanent staff work for \$8 per hour and the casual staff work for \$10 an hour. The supermarket can afford to pay their staff at most \$3000 in a 12 hour working day.

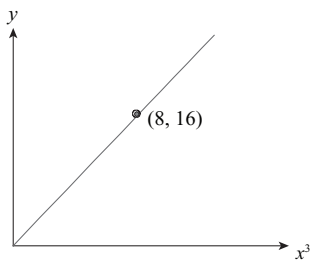
Let x be the number of permanent staff, and y be the number of casual staff that work at the local supermarket per day.

The constraint imposed by staff pay is expressed by the inequality

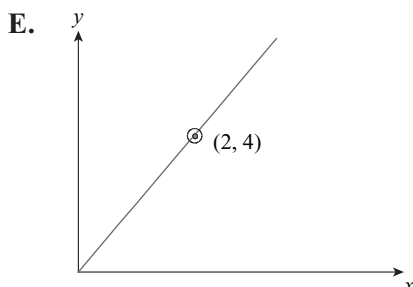
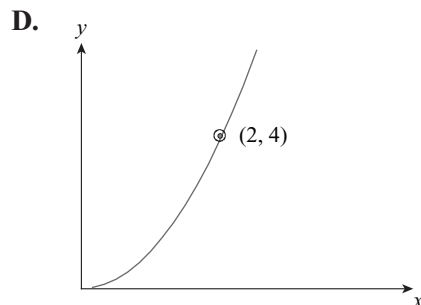
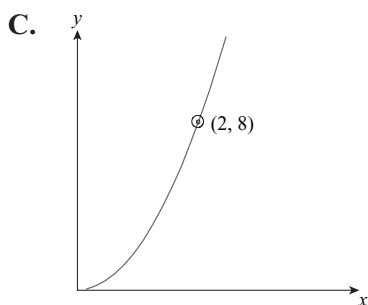
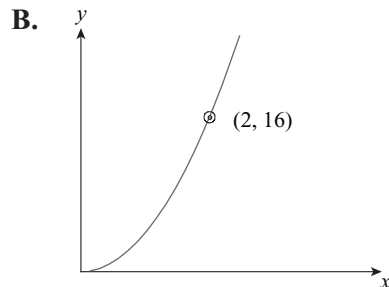
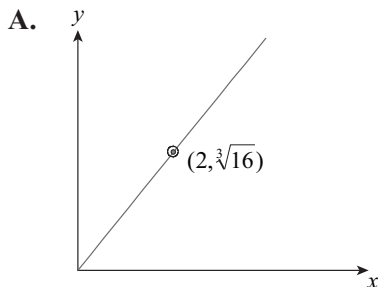
- A. $8x + 10y \leq 3000$
- B. $8x + 10y \geq 3000$
- C. $12x + 12y \leq 3000$
- D. $96x + 120y \geq 3000$
- E. $96x + 120y \leq 3000$

Question 9

The graph of y versus x^3 is shown



The graph of y versus x is:-



MODULE 4: BUSINESS-RELATED MATHEMATICS**Multiple-choice questions (9 questions)****Question 1**

Jaime has deposited her \$8000 savings in a 3-month term deposit that pays 6.2% p.a. interest. If the interest is credited at the end of the term then the amount of interest earned, to the nearest dollar, will be

- A. \$124
- B. \$125
- C. \$165
- D. \$378
- E. \$8125

Question 2

A machine bought for \$40 000 is valued at \$8200 after 6 years. The flat rate of depreciation for this machine, as a percentage correct to one decimal place, is

- A. 12.6%
- B. 13.3%
- C. 16.7%
- D. 20.5%
- E. 25.5%

Question 3

Selena has bought a property for \$376 000 and is required to pay Stamp Duty on this purchase. The price of the property means that the duty payable can be calculated using the following:

\$2560 plus 6% of the dutiable value in excess of \$115 000

The amount of stamp duty that Selena has to pay is

- A. \$9460
- B. \$15 660
- C. \$18 220
- D. \$22 560
- E. \$25 120

Question 4

Marion receives \$1210 each month as a perpetuity on an investment of \$220 000. The rate of interest per annum being credited on this investment is

- A. 5.5%
- B. 6.2%
- C. 6.6%
- D. 7.0%
- E. 7.2%

Question 5

Rajiv has invested his savings of \$15 000 in an account that pays 6.25% p.a. interest compounding each month. After three years the amount of interest that he has earned on this investment, to the nearest cent, is

- A. \$235.60
- B. \$2812.50
- C. \$2991.94
- D. \$3084.65
- E. \$3112.76

The following information relates to questions 6 and 7.

A hire-purchase plan for the purchase of a \$17 000 kitchen requires the purchaser to pay a deposit of \$1700 and then the remainder, plus interest, in 36 equal monthly instalments.

Question 6

If the credit agency providing the plan charges a flat rate of 8.5% p.a. interest then the monthly payments, to the nearest cent, will be

- A. \$498.14
- B. \$518.76
- C. \$533.38
- D. \$545.42
- E. \$547.95

Question 7

The effective rate of interest for this hire-purchase plan, to the nearest percentage, is

- A. 5 %
- B. 13 %
- C. 15 %
- D. 17 %
- E. 18 %

The following information relates to questions 8 and 9

Mike has negotiated a housing loan of \$280 000 which he will repay over 25 years. The loan attracts an interest rate of 6.75% p.a., compounding monthly, and he will make monthly repayments. Each repayment consists of an amount of principal and an amount of interest.

Question 8

The amount of principal, correct to the nearest dollar, that is repaid in the first repayment is

- A. \$360
- B. \$412
- C. \$1523
- D. \$1575
- E. \$1935

Question 9

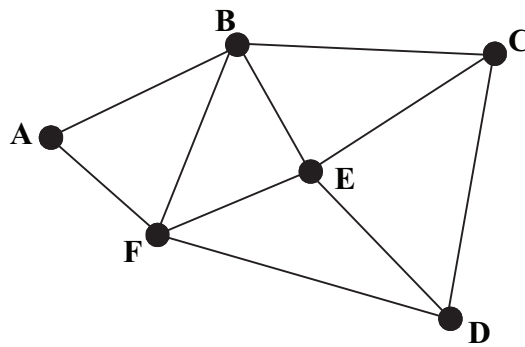
After ten years, Mike pays \$100 000 off the principal of the loan but continues repaying the same amount each month. The term of the loan will be reduced by

- A. 5 years 3 months
- B. 6 years 4 months
- C. 7 years 2 months
- D. 8 years 8 months
- E. 9 years 3 months

MODULE 5: NETWORKS AND DECISION MATHEMATICS**Multiple choice questions (9 questions)****Question 1**

A planar graph has 7 vertices and 10 edges. The graph divides the plane into

- A. 2 regions
- B. 3 regions
- C. 5 regions
- D. 8 regions
- E. 15 regions

Question 2

An Eulerian circuit is possible on the graph above if an additional edge joins two of the vertices. The edge joins vertices

- A. A to E
- B. C to D
- C. A to C
- D. A to D
- E. F to D

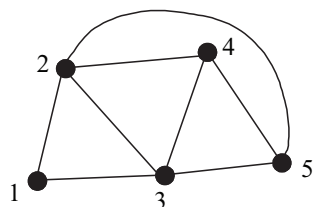
Question 3

The connectivity matrix below has been constructed for a directed network graph

		To					
		P	Q	R	S	T	U
From	P	0	0	1	1	0	0
	Q	0	0	0	1	1	0
	R	0	1	0	0	0	1
	S	0	0	0	0	1	0
	T	0	0	0	1	0	0
	U	0	0	0	0	0	0

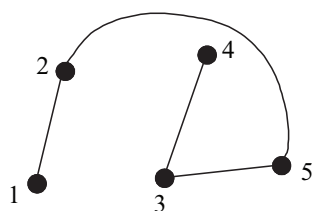
Which one of the following vertices cannot be reached from vertex R?

- A. P
- B. Q
- C. S
- D. T
- E. U

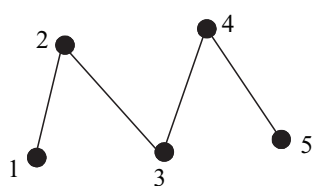
Question 4

Which one of the following is **not** a spanning tree of the graph above?

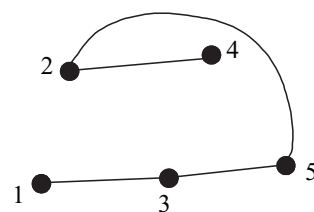
A.



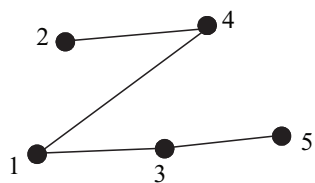
B.



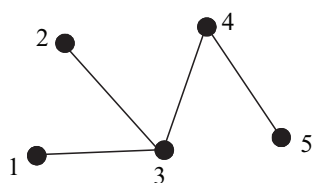
C.



D.

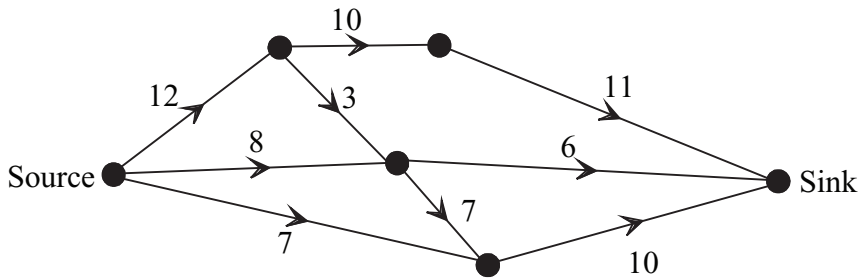


E.



Question 5

On the directed graph below, the values on the edges give the maximum flow between nodes in the direction of the arrows.



The maximum flow, from source to sink, for this graph is

- A. 25
- B. 26
- C. 27
- D. 28
- E. 31

Question 6

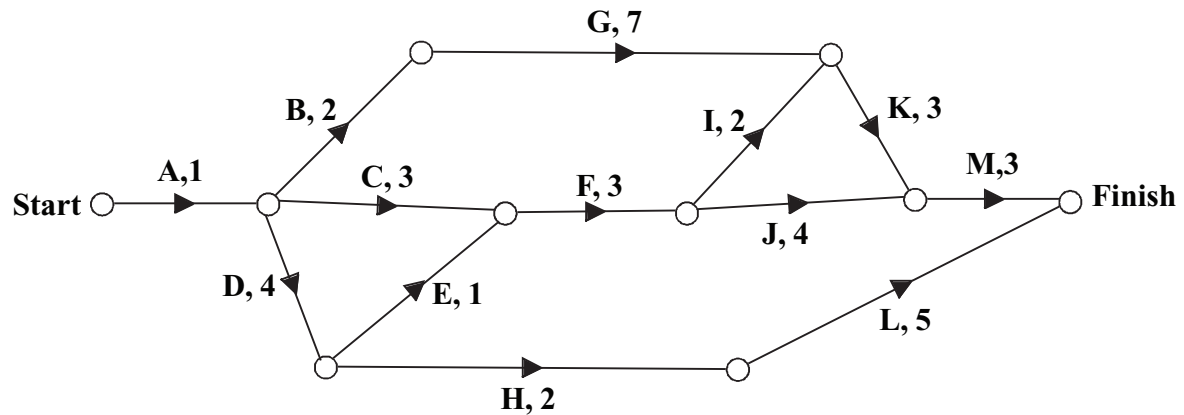
A manufacturer wants to deliver goods to each of his outlets; the delivery van making one trip starting and ending at the factory and visiting each of the outlets once only.

The route that the delivery van takes could be described as

- A. a minimum length spanning tree
- B. an Eulerian path
- C. an Eulerian circuit
- D. a Hamiltonian path
- E. a Hamiltonian circuit

The following information relates to questions 7 and 8

The graph below represents the activities (A to M) and the duration of the activities, in days, for a project.



Question 7

The critical path for the project will be

- A. ACFJM
- B. ABGKM
- C. ADHL
- D. ADEFIKM
- E. ADEFJM

Question 8

The slack time, in days, for activity L will be

- A. 0
- B. 1
- C. 2
- D. 3
- E. 5

Question 9

Five teams, P, Q, R, S and T have played in a round-robin tournament where each team plays each other team once only. The results of the games are represented by the dominance matrix M, below, where a '1' in the matrix represents a win and a '0' represents a loss.

$$\begin{array}{c}
 \text{Defeated} \\
 \begin{array}{c} \text{P} \quad \text{Q} \quad \text{R} \quad \text{S} \quad \text{T} \\
 \text{P} \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \end{bmatrix} \\
 \text{Q} \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \end{bmatrix} \\
 \text{M} = \text{R} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\
 \text{S} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
 \text{T} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}
 \end{array}
 \end{array}$$

The placings, first to last, of the teams could not be determined from the matrix M so the second-order dominance matrix, M^2 , was calculated. This is given below

$$\begin{array}{c}
 \text{Defeated} \\
 \begin{array}{c} \text{P} \quad \text{Q} \quad \text{R} \quad \text{S} \quad \text{T} \\
 \text{P} \begin{bmatrix} 0 & 0 & 1 & 2 & 2 \end{bmatrix} \\
 \text{Q} \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \end{bmatrix} \\
 \text{M}^2 = \text{R} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
 \text{S} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix} \\
 \text{T} \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \end{bmatrix}
 \end{array}
 \end{array}$$

Using M and M^2 to rank the teams, the ranking, first to last, will be

- A. PQRST
- B. PQTRS
- C. QPTSR
- D. PQTSR
- E. PTQST

MODULE 6: MATRICES**Multiple choice questions (9 questions)****Question 1**

If A is the matrix $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ then A^2 is the matrix

A. $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

E. Cannot be determined

Question 2
If the matrix $M = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}$ is multiplied by the matrix $N = \begin{bmatrix} 1 & -1 \\ 3 & 3 \\ -2 & 2 \end{bmatrix}$ then the value of the element

in the first row, second column of MN is

A. -1

B. 0

C. 1

D. 3

E. 7

Question 3

The inverse of the matrix $\begin{bmatrix} x & 4 \\ -1 & 8 \end{bmatrix}$ will not exist if x is equal to

A. -4

B. -2

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

Question 4

Matrices are to be used to solve the system of simultaneous linear equations

$$x - 3y + 16 = 0$$

and $y = 8x - 10$

The matrix equation to solve this system is

- A. $\begin{bmatrix} 1 & -3 \\ 1 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 10 \end{bmatrix}$
- B. $\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 1 & -8 \end{bmatrix} = \begin{bmatrix} 16 \\ -10 \end{bmatrix}$
- C. $\begin{bmatrix} 1 & -3 \\ 8 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -10 \end{bmatrix}$
- D. $\begin{bmatrix} 1 & -3 \\ -8 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -10 \end{bmatrix}$
- E. $\begin{bmatrix} 1 & -3 \\ -8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -10 \end{bmatrix}$

Question 5

If $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ then M is the matrix

- A. $\begin{bmatrix} -1 & 5 \\ 1 & 0 \end{bmatrix}$
- B. $\begin{bmatrix} 3 & 11 \\ 7 & 9 \end{bmatrix}$
- C. $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$
- D. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
- E. $\frac{1}{5} \begin{bmatrix} -6 & 14 \\ 1 & 6 \end{bmatrix}$

Question 6

A customer reward scheme for a shop has had a '*bonanza*' week where double bonus points were offered on sales in that week.

The total bonus points prior to *bonanza* week and the points accumulated in *bonanza* week are given for five customers in the following table.

Customer	Total points prior to <i>bonanza</i> week	Points accumulated in <i>bonanza</i> week
Dave W.	105	21
Theo N.	34	12
Janine P.	93	10
Vinh N.	68	15
Mario P.	72	5

The shop proprietor wants to use matrices to double the points accumulated in *bonanza* week and add them to the total points accumulated prior to *bonanza* week giving a total number of points for each customer.

A matrix product that will do this for the proprietor is

A. $\begin{bmatrix} 1 & 2 \end{bmatrix} \times \begin{bmatrix} 105 & 21 \\ 34 & 12 \\ 93 & 10 \\ 68 & 15 \\ 72 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 105 & 21 \\ 34 & 12 \\ 93 & 10 \\ 68 & 15 \\ 72 & 5 \end{bmatrix}$

C. $\begin{bmatrix} 105 & 21 \\ 34 & 12 \\ 93 & 10 \\ 68 & 15 \\ 72 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 105 & 21 \\ 34 & 12 \\ 93 & 10 \\ 68 & 15 \\ 72 & 5 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 2 \end{bmatrix} \times \begin{bmatrix} 105 & 21 \\ 34 & 12 \\ 93 & 10 \\ 68 & 15 \\ 72 & 5 \end{bmatrix}$

Question 7

If $\begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \times P = \begin{bmatrix} 3 & 3 \\ -2 & 0 \end{bmatrix}$ then the matrix P will be

A. $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$

E. $\frac{1}{4} \begin{bmatrix} -1 & -3 \\ 2 & 2 \end{bmatrix}$

The following information relates to questions 8 and 9

After an intense advertising campaign 38% of the available television audience watched the first episode of a television show.

Of those who watched the first episode 76% watched the second episode and of those who did not watch the first episode 15% watched the second episode.

Question 8

A transition matrix, T , that could be used to describe this situation is

A. $\begin{bmatrix} 0.76 \\ 0.15 \end{bmatrix}$ B. $\begin{bmatrix} 0.76 \\ 0.24 \end{bmatrix} + \begin{bmatrix} 0.85 \\ 0.15 \end{bmatrix}$

C. $\begin{bmatrix} 0.76 & 0.15 \\ 0.24 & 0.85 \end{bmatrix}$ D. $\begin{bmatrix} 0.76 & 0.85 \\ 0.24 & 0.15 \end{bmatrix}$

E. $\begin{bmatrix} 0.76 & 0.15 \\ 0.85 & 0.24 \end{bmatrix}$

Question 9

Assuming that the choice of television program in a particular time slot depends entirely on the choice of program in the previous week, and T is the transition matrix for this situation, the percentage of the available television audience that is watching this particular television show after five weeks can be found using

A. $T^5 \times \begin{bmatrix} 76 \\ 24 \end{bmatrix}$

B. $\begin{bmatrix} 76 & 24 \end{bmatrix} \times T^5$

C. $\left[T \times \begin{bmatrix} 38 \\ 62 \end{bmatrix} \right]^5$

D. $T^5 \times \begin{bmatrix} 38 \\ 62 \end{bmatrix}$

E. $T \times \begin{bmatrix} 38 & 0 \\ 0 & 62 \end{bmatrix}^5$

Further Mathematics Exam 1: Solutions – Multiple choice

Core : Data analysis

Question 1 Answer C

Class is a categorical variable because a category (Class 1, Class 2 or Class 3) is recorded for it. *Score on test* is a numerical variable because a number is recorded for it.

Question 2 Answer C

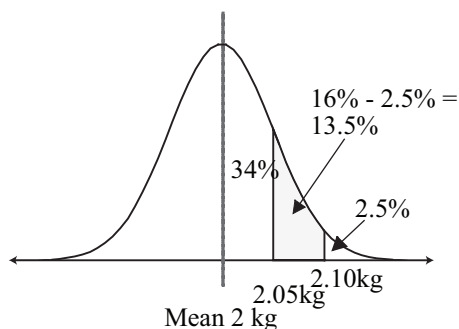
The interquartile range

= **Upper quartile** (indicated by the value of the right end of the box) – **Lower Quartile** (indicated by the value of the left end of the box)
 = 70 – 50
 = 20

Question 3 Answer D

The distribution of scores in class B is positively skewed because the boxplot is stretched to the right; the right whisker is longer than the left and the median line is towards the left of the box.

Question 4 Answer C



2.05kg is one standard deviation above the mean (16% of the bags will have weights more than 2.05kg) and 2.10 is two standard deviations above the mean (2.5% of the bags will have weights above this value). Between these 2.05kg and 2.10kg there will be $16\% - 2.5\% = 13.5\%$ of the bags.

Question 5 Answer B

Substituting the given values in the equation

$$z\text{-score} = \frac{\text{raw score} - \text{mean}}{\text{standard deviation}}$$

$$-1.25 = \frac{56 - 66}{\text{standard deviation}}$$

$$-1.25 = \frac{-10}{\text{standard deviation}}$$

$$\begin{aligned} \text{Standard deviation} &= \frac{-10}{-1.25} \\ &= 8 \end{aligned}$$

Question 6 Answer E

Of the 45 people in the 50⁺ age group, 23 of them are going to vote for the same party.

$$\frac{23}{45} \times \frac{100}{1} = 51.1\%$$

Question 7 Answer E

23 out of the 45 (51.1%) in the 50⁺ age group are going to vote for the same party.

18 out of the 42 (42.8%) of the 30 - < 50 age group are going to vote for the same party

20 out of the 33 (60.6%) of the 18 - < 30 age group are going to vote for the same party. This is more than the 50⁺ age group.

Question 8 Answer D

The right median point will be (9, 4) and the gradient of the 3-median regression line will have the same gradient as the line joining the left (2, 10) and right median points.

$$\text{Gradient} = \frac{4 - 10}{9 - 2} = \frac{-6}{7} \approx -0.857 \approx -0.9$$

Question 9 Answer E

The regression line underestimates the y -value for $x = 5$ as it is below this point.

There are the same number of points above and below the line hence the same number of positive and negative residuals.

There is a residual of -2.5 but the largest is around $+2$.

The gradient of the regression line is approx.

$$\frac{-6}{8} \approx -0.67 \text{ a value } > -1$$

Question 10 Answer B

Mar.	April	May	June	July
29	26	25	22	29

$\frac{29 + 26 + 25 + 22}{4} = 25.5$	$\frac{26 + 25 + 22 + 29}{4} = 25.5$
--------------------------------------	--------------------------------------

Centering these two values:

$$\frac{25.5 + 25.5}{2} = 25.5$$

Question 11 Answer D

3	4	Yearly average
279	173	224
283	181	231

The sales figures are divided by their yearly average and then these two values are averaged.

$$\frac{\frac{279}{224} + \frac{283}{231}}{2} = 1.2353 \text{ correct to four decimal places.}$$

Question 12 Answer A

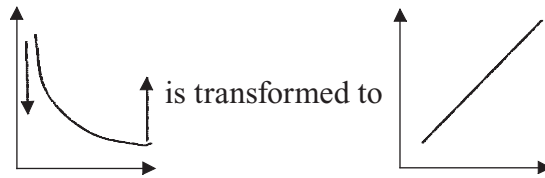
To deseasonalise data the actual data value is divided by its seasonal index.

$$\frac{190}{0.8440} = 225.1 \text{ an increase}$$

$$\frac{254}{1.1427} = 222.3 \text{ a decrease}$$

Question 13 Answer D

A $\frac{1}{y}$ transformation compresses the y scale and makes the large y -values small and the small y -values large.



Further Mathematics Exam 1: Solutions – Multiple choice

Module 1: Number Patterns

Question 1 Answer D

Arithmetic sequence

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$a = 17, d = 6, n = 12$$

$$\begin{aligned} S_{12} &= \frac{12}{2}[2 \times 17 + (12-1)6] \\ &= 6(34 + 66) \\ &= 600 \end{aligned}$$

Question 2 Answer B

$$r = 1.2, t_3 = 180, a = ?, t_n > 1000, n = ?$$

Find a first

$$ar^2 = 180$$

$$a(1.2)^2 = 180$$

$$a = \frac{180}{(1.2)^2}$$

$$\therefore a = 125$$

Find n (METHOD 1)

$$t_n > 1000, n = ?$$

$$ar^{(n-1)} > 1000$$

$$125 \times 1.2^{(n-1)} > 1000$$

$$1.2^{(n-1)} > 8$$

$$n-1 > \frac{\log_{10}(8)}{\log_{10}(1.2)}$$

$$n-1 > 11.4$$

$$n > 12.4$$

$$\therefore n = 13$$

Find n (METHOD 2)

Use sequence mode

Type in $u(n) = 125(1.2)^{(n-1)}$

Plot1	Plot2	Plot3
nMin=1		
u(n)=125*1.2^(n-1)		
u(nMin)=		
u(n)=		
u(nMin)=		
u(n)=		

Then go to TABLE

n	u(n)	
8	447.9	
9	537.48	
10	644.97	
11	773.97	
12	928.76	
13	1114.51	
14	1337.4	
u(n)=1114.512556		

Scroll down until $t_n > 1000$

Read value of n

Question 3 Answer D

Sequence 1 is decreasing and non-linear with terms alternating between positive and negative. Therefore sequence is geometric with $-1 < r < 0$

Sequence 2 is decreasing and linear, therefore sequence is arithmetic with $d < 0$

Question 4 Answer D

8, 4.8, 2.88,

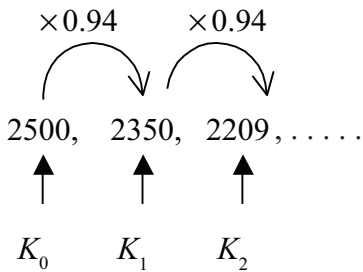
$$a = 8 \text{ and } r = \frac{2.88}{4.8} = \frac{4.8}{8} = 0.6$$

The sum of this sequence is given by

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \\ &= \frac{8}{1-0.6} \\ &= 20 \end{aligned}$$

Question 5 Answer E

Decreasing by 6% means that $r = 1 - 0.06 = 0.94$



So the difference equation is $K_n = 0.94 \times K_{n-1}$ where $K_0 = 2500$

Question 6 Answer C

The equation $W_n = 0.88 \times W_{n-1} + 50$ where $W_1 = 200$

This can be entered in the sequence mode of the calculator

$$n_{\min} = 1$$

$$u(n) = 0.88u(n-1) + 50$$

$$u(n_{\min}) = \{200\}$$

```

Plot1 Plot2 Plot3
nMin=1
u(n)=0.88u(n-1)+50
u(nMin)=200
v(n)=
v(nMin)=
w(n)=
  
```

Then go to TABLE to and scroll down until you first see $u(n) > 400$

n	$u(n)$	
17	388.64	
18	392.01	
19	394.97	
20	397.57	
21	399.86	
22	401.88	
23	403.65	

$n=22$

This occurs at $n = 22$

Question 7 Answer E

First find t_2

$$t_3 = -4t_2 - 1$$

$$51 = -4t_2 - 1$$

$$\frac{51+1}{-4} = t_2$$

$$\therefore t_2 = -13$$

Then find t_1

$$t_2 = -4t_1 - 1$$

$$-13 = -4t_1 - 1$$

$$\frac{-13+1}{-4} = t_1$$

$$\therefore t_1 = 3$$

Question 8 Answer A

An efficient method is to use the sequence mode on your calculator and enter the equation

$$u_{\min} = 1$$

$$u(n) = 2u(n-2) + u(n-1)$$

$$u(n_{\min}) = \{5, 2\}$$

enter 2nd term first $\{t_2, t_1\}$

```

Plot1 Plot2 Plot3
nMin=1
u(n)=2u(n-2)+u(n-1)
u(nMin)=5,2
v(n)=
v(nMin)=
w(n)=
  
```

Then go to the TABLE to find $u(8) = 299$

n	$u(n)$	
5	37	
6	75	
7	149	
8	299	
9	597	
10	1195	
11	2389	

$u(n)=299$

An alternative method

Working through this algebraically

Substitute $T_1 = 2$ and $T_2 = 5$

$$T_3 = 2T_1 + T_2$$

$$T_3 = 2 \times 2 + 5$$

$$= 9$$

$$T_4 = 2T_2 + T_3$$

$$T_4 = 2 \times 5 + 9$$

$$= 19$$

$$T_5 = 2T_3 + T_4$$

$$T_5 = 2 \times 9 + 19$$

$$= 37$$

$$T_6 = 2T_4 + T_5$$

$$T_6 = 2 \times 19 + 37$$

$$= 75$$

$$T_7 = 2T_5 + T_6$$

$$T_7 = 2 \times 37 + 75$$

$$= 149$$

$$T_8 = 2T_6 + T_7$$

$$T_8 = 2 \times 75 + 149$$

$$= 299$$

Question 9 Answer BFor a Fibonacci sequence, $f_{10} + f_{11} + f_{12}$

$$\therefore f_{11} = f_{12} - f_{10}$$

$$= 555 - 212$$

$$= 343$$

Further Mathematics Exam 1: Solutions – Multiple choice

Module 2: Geometry & Trigonometry Solutions

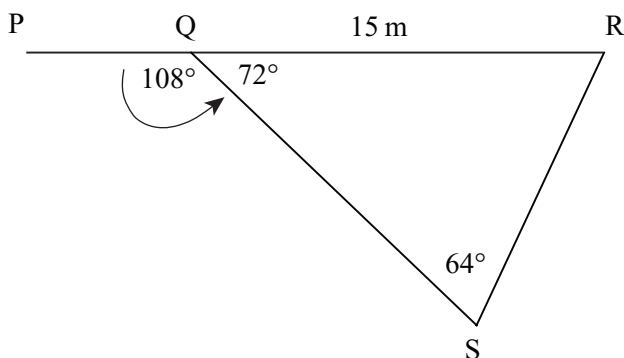
Question 1 Answer A

$$\angle QSR + \angle QRS = \angle PQS$$

$$64^\circ + \angle QRS = 108^\circ$$

$$\angle QRS = 44^\circ$$

Question 2 Answer E



$$\angle SQR = 180^\circ - 108^\circ = 72^\circ$$

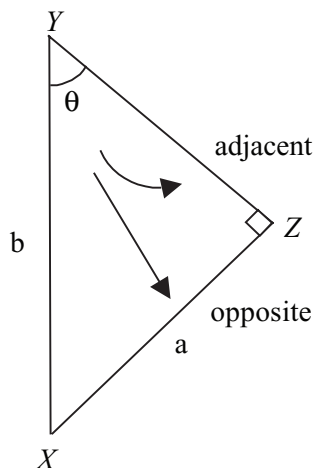
Using the sine rule

$$\frac{RS}{\sin 72^\circ} = \frac{15}{\sin 64^\circ}$$

$$RS = \frac{15 \times \sin 72^\circ}{\sin 64^\circ}$$

$$= 15.872 \text{ m}$$

Question 3 Answer E



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{XZ}{YZ}$$

Using Pythagoras

$$YZ^2 + XZ^2 = XY^2$$

$$YZ^2 = XY^2 - XZ^2$$

$$YZ^2 = b^2 - a^2$$

$$YZ = \sqrt{b^2 - a^2}$$

$$\therefore \tan \theta = \frac{a}{\sqrt{b^2 - a^2}}$$

Question 4 Answer E

Using Heron's Formula

$$s = \frac{3x + 5x + 6x}{2} = 7x$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{7x(7x-3x)(7x-5x)(7x-6x)}$$

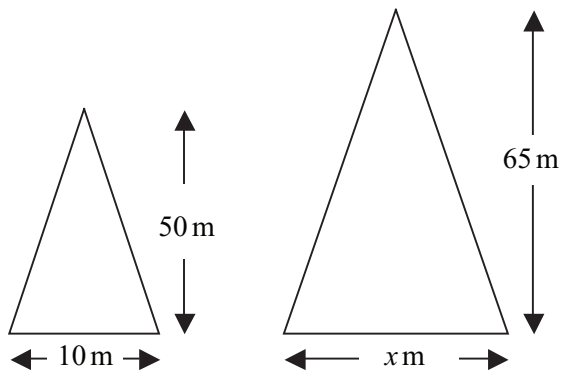
$$= \sqrt{7x(4x)(2x)(x)}$$

$$= \sqrt{56x^4}$$

$$= \sqrt{56}x^2$$

Question 5 Answer B

Using similar triangles

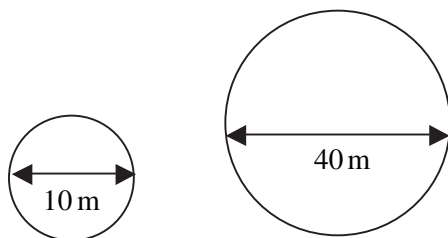


$$\frac{x}{10} = \frac{65}{50}$$

$$x = \frac{65 \times 10}{50}$$

$$x = 13 \text{ metres}$$

Question 6 Answer C



Length ratio is 1 : 4

Area ratio is $1^2 : 4^2$

$$= 1 : 16$$

Therefore area is 16 times larger.

Question 7 Answer D

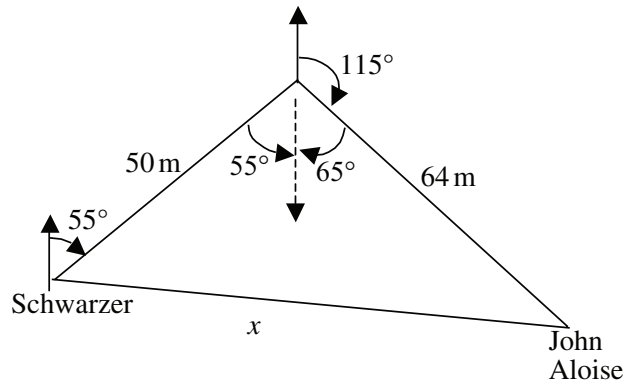
$$\text{Area of Trapezium} = \frac{1}{2}(a + b)h$$

$$240 = \frac{1}{2}(20 + 28)h$$

$$240 = 24h$$

$$\therefore h = \frac{240}{24} = 10 \text{ cm}$$

Question 8 Answer E



Using cosine rule

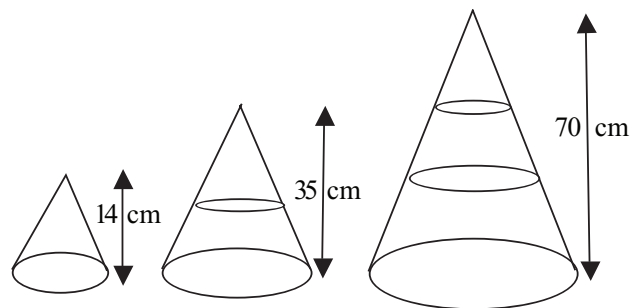
$$x^2 = 50^2 + 64^2 - 2(50)(64)\cos 120^\circ$$

$$x^2 = 9796$$

$$x = 98.97$$

$$x \approx 99 \text{ m}$$

Question 9 Answer A



Length ratio is 14 : 35 : 70

$$= 2 : 5 : 10$$

Area ratio is $2^2 : 5^2 : 10^2$

The ratio of the olive : bottle green is

$$25 - 4 : 100 - 25$$

$$21 : 75$$

$$= 7 : 25$$

Further Mathematics Exam 1: Solutions – Multiple choice

Module 3: Graphs and Relations

Question 1 Answer C

The vertical line has no gradient. Every point on this line has the same x value. Since the point $(5, 2)$ is on the line then the equation is given by $x = 5$.

Question 2 Answer D

Transform the equation to make y the subject

$$3x + 5y = 30$$

$$5y = 30 - 3x$$

$$y = 6 - \frac{3}{5}x$$

This means that the gradient is $-\frac{3}{5}$, so the graph is decreasing

i.e. as x increases, y decreases or conversely as x decreases, y increases $\therefore$ A is true

When $x = 0$, $y = 6 \therefore$ B is true

When $y = 0$, $x = 10 \therefore$ C is true

The gradient is $-\frac{3}{5}$, not $-\frac{5}{3}$ so D is false

Substituting $x = 5$ in equation gives $y = 3 \therefore$ E is true

Question 3 Answer B

Let s = the cost of one scarf and

h = the cost of one hat

$$3s + 2h = 60 \quad - \quad (1)$$

$$2s + 5h = 73 \quad - \quad (2)$$

To eliminate s

$$3 \times (2) - 2 \times (1)$$

$$6s + 15h = 219$$

$$6s + 4h = 120$$

$$11h = 99$$

$$\therefore h = \$9$$

Question 4 Answer D

\$22 is charged for 2 hours.

Therefore Betty paid $\$48 - \$22 = \$26$.

\$26 is charged for 3 to 4 hours of parking.

Question 5 Answer D

For 0 – 20 minutes

$$m \approx \frac{180 - 20}{20} = 8^\circ\text{C} / \text{min}$$

For 20 – 40 minutes

$$m \approx \frac{160 - 180}{20} = -1^\circ\text{C} / \text{min}$$

For 40 – 60 minutes

$$m \approx \frac{165 - 160}{20} = 0.25^\circ\text{C} / \text{min}$$

For 60 – 80 minutes

$$m \approx \frac{50 - 165}{20} = -5.75^\circ\text{C} / \text{min}$$

For 80 – 100 minutes

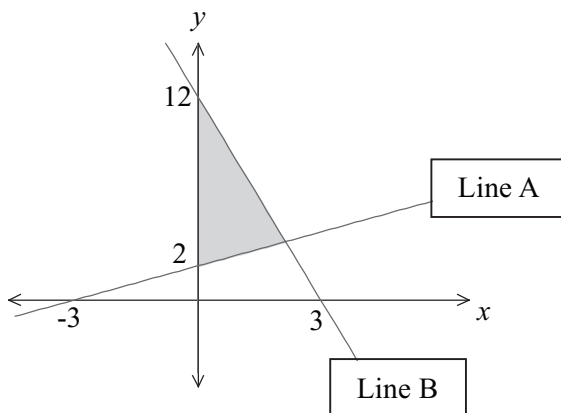
$$m \approx \frac{25 - 50}{20} = -1.25^\circ\text{C} / \text{min}$$

The steepest negative gradient occurs between 60 to 80 minutes.

Question 6 Answer D

Vertex	$2x + 4y$
A(0, 8)	$2 \times 0 + 4 \times 8 = 32$
B(4, 6)	$2 \times 4 + 4 \times 6 = 32$
C(6, 4)	$2 \times 6 + 4 \times 4 = 28$
D(8, 0)	$2 \times 8 + 4 \times 0 = 16$
E(0, 6)	$2 \times 0 + 4 \times 6 = 24$

The maximum occurs along the boundary line joining points A and B.

Question 7 Answer A**Line A**

Passes through points $(-3, 0)$ and $(0, 2)$

$$\text{Gradient} = \frac{2}{3}, \text{y-intercept} = 2$$

$$\therefore \text{equation is } y = \frac{2}{3}x + 2$$

$$3y = 2x + 6$$

$$3y - 2x = 6$$

Test point $(0, 3)$ is in the required region

$$3 \times 3 - 2 \times 0 \geq 6$$

$$\therefore 3y - 2x \geq 6$$

defines required region.

This eliminates D

Line B

Passes through points $(0, 12)$ and $(3, 0)$

$$\text{Gradient} = -\frac{12}{3} = -4, \text{y-intercept} = 12$$

$$\therefore \text{equation is } y = -4x + 12$$

$$y + 4x = 12$$

Test point $(0, 3)$ is in the required region

$$3 + 4 \times 0 \leq 12$$

$$\therefore y + 4x \leq 12$$

defines required region.

This eliminates B and C.

Since the feasible region shows all x values to be positive then $x \geq 0$. This eliminates E.

Question 8 Answer EPermanent Staff

For a 12 hour day each of the permanent staff will be paid $12 \times \$8 = \96 per day

If x permanent staff are working for the day then a total of $\$96x$ is paid.

Casual Staff

For a 12 hour day each of the casual staff will be paid $12 \times \$10 = \120 a day

If y casual staff are working for the day then a total of $\$120y$ is paid.

Total Pay

This gives a total of $96x + 120y$ dollars per day.

The supermarket can afford to pay *at most* 3000 per day i.e. less than or equal to \$3000

$$\text{So } 96x + 120y \leq 3000$$

Question 9 Answer B

The graph of y against x^3 is linear and passes through the origin so the equation of the relation is $y = kx^3$ where k represents the gradient of the graph.

$$\text{Gradient} = \frac{16}{8} = 2 \quad \therefore k = 2$$

The equation of the relation is $y = 2x^3$.

The graph of y against x produces a non-linear graph. This eliminates A and E.

$$\text{When } x = 2, y = 2 \times 2^3 = 2 \times 8 = 16$$

This means the point $(2, 16)$ is on the graph.

Further Mathematics Exam 1: Solutions – Multiple choice

Module 4: Business-related mathematics

Question 1 Answer A

$$\text{Monthly interest} = \frac{6.2}{12}\%$$

Using the simple interest formula:

$$\begin{aligned}\text{Interest earned} &= \frac{Prt}{100} = \frac{\$8000 \times \frac{6.2}{12} \times 3}{100} \\ &= \$124\end{aligned}$$

Question 2 Answer B

$$\begin{aligned}\text{Depreciation} &= \$40\,000 - \$8200 \\ &= \$31\,800 \text{ in six years.}\end{aligned}$$

Flat rate depreciation means that the depreciation is the same each year.

$$\begin{aligned}\text{Depreciation per year} &= \frac{\$31\,800}{6} \\ &= \$5300\end{aligned}$$

As a percentage of the purchase price this is

$$\frac{5300}{40000} \times \frac{100}{1} = 13.25\% \approx 13.3\%$$

Question 3 Answer C

Stamp duty

$$\begin{aligned}&= \$2560 + \frac{6}{100} \times (\$376\,000 - \$115\,000) \\ &= \$2560 + \$15\,660 \\ &= \$18\,220\end{aligned}$$

Question 4 Answer C

\$1210 is the interest earned per month on the investment of \$220 000.

$$\begin{aligned}\text{Interest earned per year} &= 12 \times \$1210 \\ &= \$14\,520\end{aligned}$$

As a percentage this is

$$\frac{14\,520}{220\,000} \times \frac{100}{1} = 6.6\%$$

Question 5 Answer D

Using the compound interest formula the amount in the account: $A = PR^n$

where

$$P = \$15\,000$$

$$R = 1 + \frac{\frac{6.25}{12}}{100} = 1.005208 \dots \text{leave on calculator}$$

$$n = 3 \times 12 = 36 \text{ months}$$

$$A = 15\,000 \times (1.005208\dots)^{36}$$

$$\approx 18\,084.652$$

So the interest earned

$$= \$18\,084.65 - \$15\,000$$

$$= \$3084.65 \text{ to the nearest cent.}$$

Question 6 Answer C

$$\begin{aligned}\text{Amount owed} &= \$17\,000 - \$1700 \\ &= \$15\,300\end{aligned}$$

Interest that will accumulate over three years

$$= \frac{\$15\,300 \times 8.5 \times 3}{100} = \$3901.50$$

$$\begin{aligned}\text{Total owing} &= \$15\,300 + \$3901.50 \\ &= \$19\,201.50\end{aligned}$$

$$\begin{aligned}\text{Monthly payment} &= \frac{\$19\,201.50}{36} = \$533.38 \text{ to} \\ &\text{the nearest cent.}\end{aligned}$$

Question 7 Answer D

Using the formula.

Effective rate of interest

$$\approx \frac{2n}{n+1} \text{ flat rate of interest}$$

Effective rate of interest

$$= \frac{2 \times 36}{36+1} \times 8.5$$

$$= \frac{72}{37} \times 8.5$$

$$= 16.54\%$$

$$\approx 17\%$$

Question 8 Answer A

Use the TVM solver to find the monthly payment:

```
N=300
I%=6.75
PV=280000
PMT=-1934.5522...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

The monthly payment is \$1934.55

From this we can deduct the interest earned on the loan in the first month:

$$I = \frac{\$280\,000 \times \frac{6.75}{12} \times 1}{100} = \$1575$$

The amount of principal repaid in the first month is \$359.55 $\approx$ \$360

Alternatively:

Use the 'sum of the principal paid over a specified period' calculator on the FINANCE menu:

```
CALC VARS
6:tvM_FV
7:nPV(
8:irr(
9:bal(
10:ΣPrn(
A:ΣInt(
B:ΣNom(
```

```
ΣPrn(1,1)
-359.5522706
```

Question 9 Answer D

Firstly find the principal remaining after 10 years.

```
N=120
I%=6.75
PV=280000
PMT=-1934.5522...
FV=-218615.8419
P/Y=12
C/Y=12
PMT:END BEGIN
```

Deduct from the principal remaining the \$100 000 and make this the present value (PV) and the future value (FV) equal to 0.

```
N=120
I%=6.75
PV=118615.8419
PMT=-1934.5522...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

Now SOLVE for N.

```
N=75.4036633
I%=6.75
PV=118615.8419
PMT=-1934.5522...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

It will take 76 more months to repay the loan; a total of $10 \times 12 + 76 = 196$ months instead of 300 months (25 years).

The term is reduced by 104 months
= 8 years, 8 months.

Further Mathematics Exam 1: Solutions – Multiple choice

Module 5: Networks and decision mathematics

Question 1 Answer C

Euler's formula for planar graphs is

$$v + f = e + 2$$

where v is the number of vertices, f is the number of 'faces' or regions and e is the number of edges.

Substituting:

$$\begin{aligned} 7 + f &= 10 + 2 \\ f &= 5 \end{aligned}$$

Question 2 Answer B

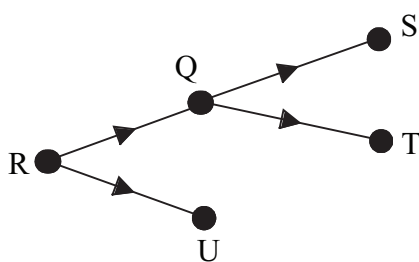
An Eulerian circuit exists when the degree of all the vertices is even. In this graph all the vertices are of even degree except C and D (both degree 3).

An edge joining vertex C to vertex D would make all vertices of even degree and so an Eulerian circuit would exist.

Question 3 Answer A

There is an edge going from vertex R to vertices Q and U

From vertex Q there are edges going to vertices S and T but there are no edges going from vertex U.



So S, Q, T and U can be reached from R but not vertex P.

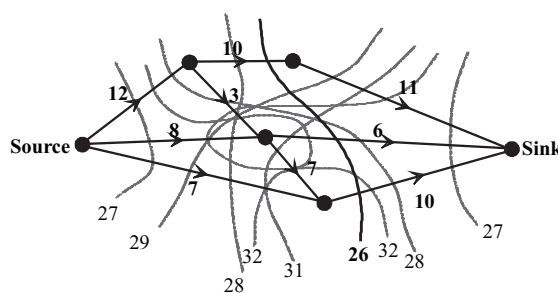
Question 4 Answer D

A spanning tree is a subset of the original graph which connects all the vertices but contains no circuits.

All the answer graphs are connected, contain no circuits and contain all the vertices but the graph in Answer D has an extra edge going from vertex 4 to vertex 1 which means it is not a subset of the original graph.

Question 5 Answer B

The maximum flow from source to sink is the value of the minimum cut



All possible cuts are shown and the minimum is 26.

Question 6 Answer E

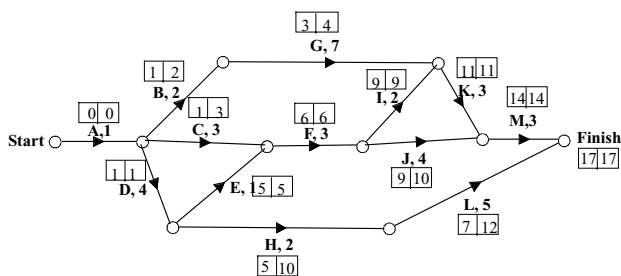
A Hamiltonian circuit is a path that visits every vertex (outlet) exactly once, starting and ending at the same vertex (the factory).

Question 7 Answer D

The critical path of a project is the sequence of activities that is the longest throughout the network.

Activities on the critical path have the same *earliest start time* and *latest start time*. (These are the figures in the rectangle on the diagram below.)

A forward and backward scan will identify the critical path.



The critical path is **ADEFIKM**

Question 8 Answer E

The slack time is the difference between the *earliest start time* and *latest start time*. For activity **L** this is $12 - 7 = 5$ days

Question 9 Answer D

The dominance vector for M (found by adding the values in each of the rows of M) is

$$\begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \\ 2 \end{bmatrix} \text{ which gives the number of first-order wins}$$

for the teams but does not give a ranking for the teams.

The dominance vector for $M + M^2$ is

$$\begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 4 \\ 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 2 \\ 3 \\ 6 \end{bmatrix} \begin{matrix} P \\ Q \\ R \\ S \\ T \end{matrix}$$

Team P had the highest number of wins (first and second order) followed by Q then T, then S and last was R

Further Mathematics Exam 1: Solutions – Multiple choice

Module 6 : Matrices

Question 1 Answer B

$$\begin{aligned}
 A^2 &= \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times 1 + 2 \times 0 & 1 \times 2 + 2 \times -1 \\ 0 \times 1 + -1 \times 0 & 0 \times 2 + -1 \times -1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
 \end{aligned}$$

Alternatively use the calculator.

```

MATRIX[A] 2 ×2
[[ 1  2 ]
 [ 0 -1 ]]

```

```

[A]^2
[[ 1  0 ]
 [ 0  1 ]]

```

```

[A]^-1*[B]
[[ 1  1 ]
 [ 1 -1 ]]

```

Question 2 Answer A

The element in the first row, second column of MN

= 1st row of M × 2nd column of N

= $2 \times -1 + 1 \times 3 + -1 \times 2$

= -1

Question 3 Answer C

The inverse will not exist if the determinant is zero.

ie. $x \times 8 - -1 \times 4 = 0$

$8x + 4 = 0$

$8x = -4$

$x = -\frac{1}{2}$

Question 4 Answer E

Re-arranging the equations to 'standard form'.

$x - 3y = -16$

$-8x + y = -10$

gives the equations

$$\begin{bmatrix} 1 & -3 \\ -8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -10 \end{bmatrix}$$

Question 5 Answer D

$$\begin{aligned}
 M &= \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \\
 &= \frac{1}{3 \times 1 - 2 \times -1} \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \\
 &= \frac{1}{5} \begin{bmatrix} 5 & 5 \\ 5 & -5 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
 \end{aligned}$$

Alternatively use the calculator.

```

MATRIX[A] 2 ×2
[[ 3 -1 ]
 [ 2  1 ]]

```

```

MATRIX[B] 2 ×2
[[ 2  4 ]
 [ 3  1 ]]

```

Question 6 Answer D

The points matrix is a 5×2 matrix so this eliminated answers A, B and E because the multiplications are not possible. Answer C will not give a total for each of the customers.

Question 7 Answer B

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \times P = \begin{bmatrix} 3 & 3 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So P is equal to the inverse of $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$$P = \frac{1}{1 \times 3 - 1 \times 2} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

Question 8 Answer C

Of those who watched the first episode 76% watched the second episode and 24% watched another show. Of those who did not watch the first episode 15% watched the second episode and 85% watched another show.

The transition matrix, T , will be formed from the table:

		1 st episode	
		<i>Watch</i>	<i>Other</i>
2 nd episode	<i>Watch</i>	0.76	0.15
	<i>Other</i>	0.24	0.85

Question 9 Answer D

The initial state matrix will be $\begin{bmatrix} 38 \\ 62 \end{bmatrix}$ using the percentages of the available audience initially watching the show.

If the transition matrix is T then after five weeks the expected percentage of the available audience still watching the television show will be given by the element in the first row of the matrix

$$T^5 \times \begin{bmatrix} 38 \\ 62 \end{bmatrix}$$

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:
$$z = \frac{x - \bar{x}}{s_x}$$

least squares line:
$$y = a + bx \quad \text{where } b = r \frac{s_y}{s_x} \quad \text{and} \quad a = \bar{y} - b\bar{x}$$

residual value:
$$\text{residual value} = \text{actual value} - \text{predicted value}$$

seasonal index:
$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

Module 1: Number patterns

arithmetic series:
$$a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} (a + l)$$

geometric series:
$$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$$

infinite geometric series:
$$a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, |r| < 1$$

Module 2: Geometry and trigonometry

area of a triangle:
$$\frac{1}{2} bc \sin A$$

Heron's formula:
$$A = \sqrt{s(s - a)(s - b)(s - c)} \quad \text{where } s = \frac{1}{2}(a + b + c)$$

circumference of a circle:
$$2\pi r$$

area of a circle:
$$\pi r^2$$

volume of a sphere:
$$\frac{4}{3} \pi r^3$$

surface area of a sphere:
$$4\pi r^2$$

volume of a cone:
$$\frac{1}{3} \pi r^2 h$$

volume of a cylinder:
$$\pi r^2 h$$

volume of a prism:
$$\text{area of base} \times \text{height}$$

volume of a pyramid:
$$\frac{1}{3} \text{ area of base} \times \text{height}$$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET