

Mathematical Association of Victoria
MATHEMATICAL METHODS
2008 Trial written examination 2 – Solutions – Multiple choice

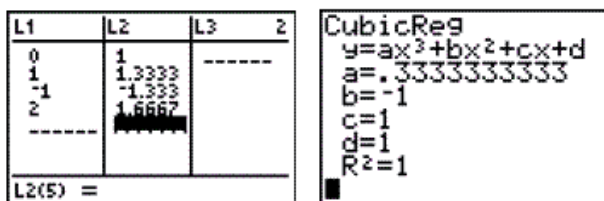
SECTION 1
Solutions

1. D 2. E 3. A 4. C 5. C 6. B
 7. E 8. D 9. A 10. B 11. B 12. A
 13. D 14. E 15. B 16. E 17. A 18. C
 19. A 20. B 21. B 22. A

Question 1

Answer D

Enter data into the calculator and choose cubic regression.



The exact answer is required.

$$y = \frac{1}{3}x^3 - x^2 + x + 1$$

$$3y = x^3 - 3x^2 + 3x + 3$$

Question 2

Answer E

$h(x)$ has factors x , $x(x - b)$ and $(x - c)$.

The coefficient of x^4 is negative.

Since $a > 0$, the rule is

$$h(x) = -ax^2(x - b)(x - c).$$

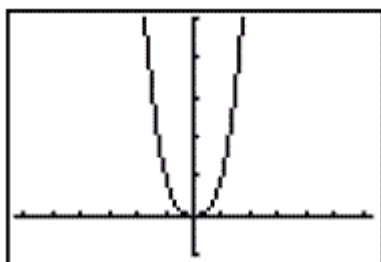
Question 3

Answer A

$$f(g(x)) = e^{3\log_e|x|} = e^{\log_e|x|^3} = |x^3|$$

The domain of $f(g(x))$ is the same as the domain of $g(x) : \mathbb{R} \setminus \{0\}$.

The range is $\mathbb{R}^+$.



Question 4**Answer C**

$f: (-\infty, 3] \rightarrow R$ where $f(x) = (x - 3)^{\frac{2}{3}} + 1$

Let $y = (x - 3)^{\frac{2}{3}} + 1$

Inverse swap x and y

$$x = (y - 3)^{\frac{2}{3}} + 1$$

$$(y - 3)^{\frac{2}{3}} = x - 1$$

$y - 3 = -(x - 1)^{\frac{3}{2}}$, take the negative square root

$$y = 3 - (x - 1)^{\frac{3}{2}}$$

$f: [1, \infty) \rightarrow R$, where $f^{-1}(x) = 3 - (x - 1)^{\frac{3}{2}}$

Question 5**Answer C**

At point of intersection:

$$2x^2 + mx = x - 8$$

$$2x^2 + (m - 1)x + 8 = 0$$

There will be exactly one point of intersection when the discriminant, Δ , equals zero.

$$\Delta = 0$$

$$(m - 1)^2 - 4 \times 2 \times 8 = 0$$

$$m^2 - 2m - 63 = 0$$

$$(m - 9)(m + 7) = 0$$

$$m = 9 \text{ or } m = -7$$

Question 6**Answer B**

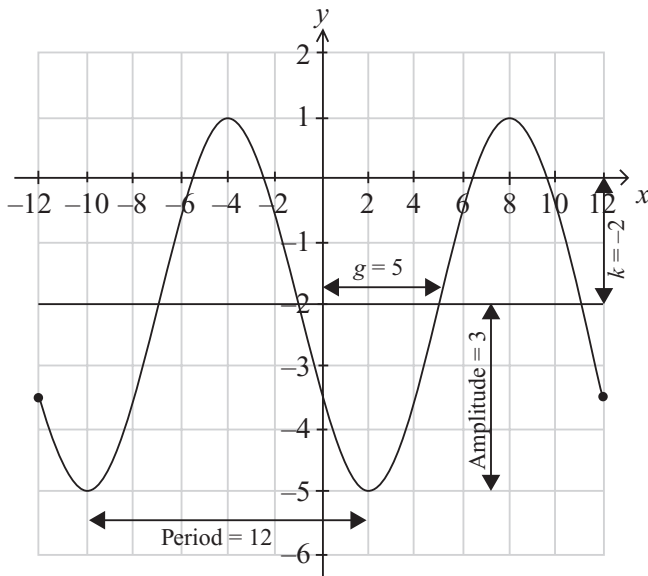
$$-h(2x - 4) = -h(2(x - 2))$$

The transformation from $h(x)$ to $-h(2(x - 2))$ involves

A reflection in the x -axis, a dilation by a scale factor of $\frac{1}{2}$ from the y -axis and a translation 2 units right.

Question 7

Answer E



The rule will be of the form

$$f(x) = a \sin(n(x - \epsilon)) + k.$$

Amplitude = 3, therefore $a = 3$.

Translation down 2 units, therefore $k = -2$.

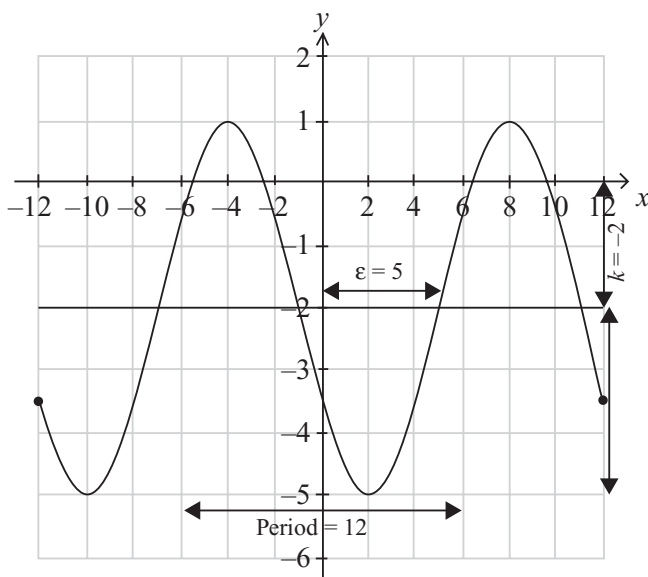
Phase shift, $\epsilon = 5$.

$$\text{Period} = \frac{2\pi}{n}$$

$$12 = \frac{2\pi}{n}$$

$$n = \frac{2\pi}{12}$$

$$n = \frac{\pi}{6}$$



Question 8**Answer D**

$f: [-\pi, \pi] \rightarrow \mathbb{R}$ where $f(x) = -4\tan(3x)$

Asymptotes $x = \frac{(2k+1)\pi}{6}, k \in \mathbb{Z}$

For the given domain there are 6 asymptotes.

Question 9**Answer A**

Reflection in the y -axis of $y = e^x$ to $y = e^{-x}$

Dilation by a factor of $\frac{1}{2}$ from the y -axis $y = e^{-x}$ to $y = e^{-2x}$

A translation 1 unit left $y = e^{-2(x+1)}$

Question 10**Answer B**

$$e^{2x} + be^x + 1 = 0$$

Let $e^x = p$

$$p^2 + bp + 1 = 0$$

$\Delta < 0$ for no real solutions

$$b^2 - 4 = 0$$

Hence $\{b : -2 < b < 2\}$

Also there is no solution at $b = 2$ as

$$e^{2x} + 2e^x + 1 = (e^x + 1)^2 = 0$$

$$e^x \neq -1$$

Thus $\{b : -2 < b \leq 2\}$

Question 11**Answer B**

$$f(x) = \begin{cases} x^2 + 4x + a & \text{for } x \geq 0 \\ bx + c & \text{for } x < 0 \end{cases}$$

For $f(x)$ to be differentiable at $x = 0$, $f(x)$ has to be continuous at $x = 0$.

$$x^2 + 4x + a = bx + c$$

Hence at $x = 0$, $a = c$

$$\text{Also } \lim_{x \rightarrow 0^-} f'(x) = \lim_{x \rightarrow 0^+} f'(x)$$

$$2x + 4 = b$$

$$\text{At } x = 0 \quad b = 4$$

Question 12**Answer A**

$$f(x+h) \approx hf'(x) + f(x)$$

$$f(x+h) - f(x) \approx hf'(x)$$

$$\approx -0.1 \times \frac{2}{2x+1}$$

$$\approx -0.1 \times \frac{2}{3} = -\frac{1}{15}$$

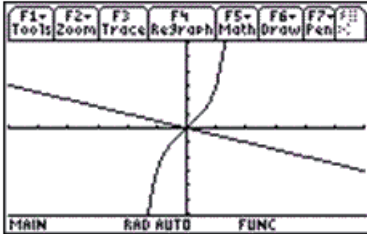
Question 13**Answer D**

$$y = x^5 + 2x$$

$$\frac{dy}{dx} = 5x^4 + 2 = 2 \text{ at } x = 0$$

The equation of the normal is

$$y = -\frac{1}{2}x.$$

**Question 14****Answer E**

$$\frac{x^2}{y} = 1, y = x^2$$

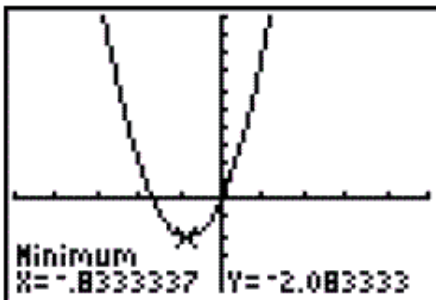
Substitute $y = x^2$ into p

$$p = 5x + 3y = 5x + 3x^2$$

$$\frac{dp}{dx} = 5 + 6x = 0 \text{ or } x = -\frac{b}{2a} = -\frac{5}{6}$$

$$x = -\frac{5}{6}$$

p has a minimum value when $x = -\frac{5}{6}$



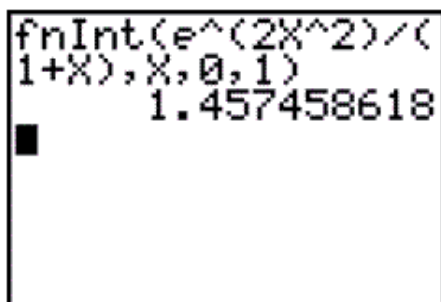
Question 15**Answer B**

$$x = \sqrt{1-t}$$

$$v = \frac{dx}{dt} = -\frac{1}{2\sqrt{1-t}}$$

$$a = \frac{dv}{dt} = -\frac{1}{4(1-t)^{\frac{3}{2}}}$$

Question 16**Answer E**



fnInt(e^(2X^2)/(1+X),X,0,1)
1.457458618

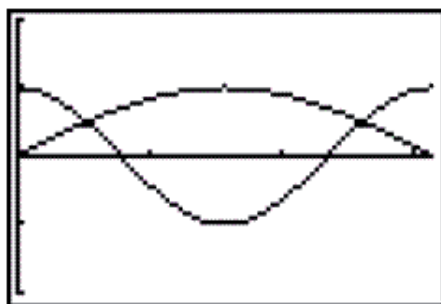
$$\int_0^1 \left(\frac{e^{2x^2}}{1+x} \right) dx \approx 1.5$$

Question 17**Answer A**

$y = \sin(x)$ is the top curve and $y = \cos(2x)$ is the bottom curve.

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) \text{ and } \sin\left(\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{3}\right)$$

$$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin(x) - \cos(2x)) dx$$



Question 18**Answer C**

For a probability density function,

$$\int_{-\infty}^{\infty} f(t) dt = 1.$$

$$0 + \int_{-\pi}^{\pi/2} k dt = 1$$

$$\left[kt \right]_{-\pi}^{\pi/2} = 1$$

$$\left[\frac{\pi}{2} k - (-\pi k) \right] = 1$$

$$\frac{3\pi}{2} k = 1$$

$$k = \frac{2}{3\pi}$$

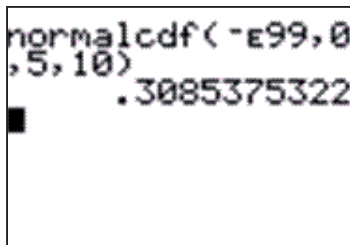
Question 19**Answer A**

Let X be the percentage increase in the value of stocks.

$$X \sim N(\mu = 5, \sigma^2 = 10^2)$$

Stocks decrease in value if $X < 0$.

$$\Pr(X < 0) \approx 0.31$$

**Question 20****Answer B**

This is a problem of selection without replacement. Let S denote dialling Stewart's phone number.

$$\Pr(S', S', S) = \frac{7}{8} \times \frac{6}{7} \times \frac{1}{6}$$

Note from this pattern that the probability that the first, second, third, ..., eighth number dialled is Stewart's number, is $\frac{1}{8}$ each time.

Question 21**Answer B**

$$\sum p(x) = 1$$

$$h + h^2 + \frac{1}{2} - h = 1$$

$$h^2 = \frac{1}{2}$$

$$h = \frac{1}{\sqrt{2}} \text{ (Reject negative solution as } h \in [0,1])$$

$$\mu = \sum x p(x)$$

$$= 0 + \left(1 \times \left(\frac{1}{\sqrt{2}} \right)^2 \right) + \left(\sqrt{2} \times \left(\frac{1}{2} - \frac{1}{\sqrt{2}} \right) \right)$$

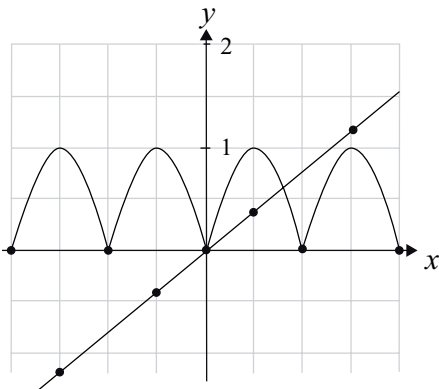
$$= \frac{1}{2} + \frac{\sqrt{2}}{2} - 1$$

$$= \frac{\sqrt{2}}{2} - \frac{1}{2}$$

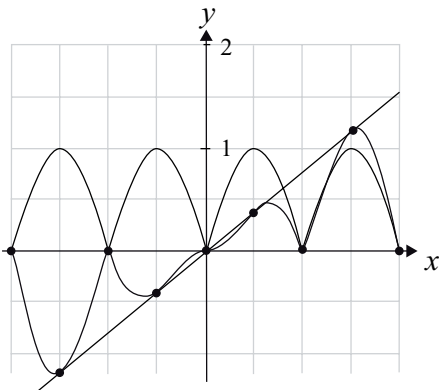
$$= \frac{\sqrt{2} - 1}{2}$$

Question 22**Answer A**

Let the maximum value of g be 1 unit. For graphs of products of functions, focus on the points where either function has the value of either 1 or 0. Place a dot on the graph for the product of these points.

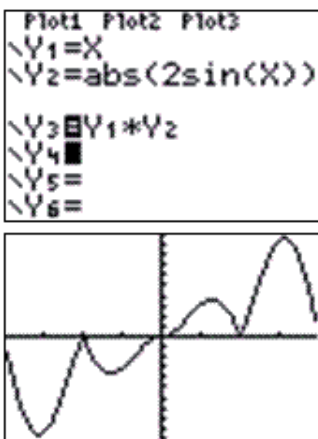


Join the dots



Alternatively

The graph of $y = f(x)$ could be $y = x$ and the graph of $y = g(x)$ could be $y = |2\sin(x)|$.



The graph of the product is closest to graph A.

Mathematical Association of Victoria
MATHEMATICAL METHODS
2008 Trial written examination 2 – Solutions – Extended Answer

SECTION 2**Question 1**

a. Solve $h^2 + \left(\frac{b}{2}\right)^2 = b^2$ for h

$$h^2 = b^2 - \frac{b^2}{4}$$

$$= \frac{3b^2}{4}$$

$$h = \frac{\sqrt{3}b}{2}$$

[1M]

SA = area of the 2 triangles + 3 rectangles

$$bh + 3lb = 120$$

$$\frac{\sqrt{3}b^2}{2} + 3lb = 120$$

[1M]

$$3lb = 120 - \frac{\sqrt{3}b^2}{2}$$

$$l = \frac{40}{b} - \frac{\sqrt{3}b}{6}$$

[1M]

$$l = \frac{240 - \sqrt{3}b^2}{6b}, 3 \leq b \leq 9 \text{ as required}$$

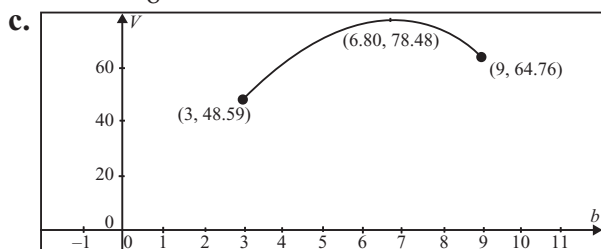
b. $V = \text{area of the triangle} \times \text{length}$

[1M]

$$= \frac{b}{2} \times \frac{\sqrt{3}b}{2} \times \frac{240 - \sqrt{3}b^2}{6b}$$

[1M]

$$= \frac{b(80\sqrt{3} - b^2)}{8}, 3 \leq b \leq 9 \text{ as required}$$

Correct shape **[1A]**Correct turning point (6.80, 78.48) **[1A]**

Correct end points with closed circles

(3, 48.59) and (9, 64.76) **[1A]**

d. The maximum occurs at the turning point not an end point.

$$\frac{dV}{db} = 10\sqrt{3} - \frac{3b^2}{8}$$

$$\text{Solve } 10\sqrt{3} - \frac{3b^2}{8} = 0 \text{ for } b \quad [1M]$$

$$-\frac{3b^2}{8} = -10\sqrt{3}$$

$$b^2 = \frac{80\sqrt{3}}{3} \quad [1M]$$

$$b = \frac{4\sqrt{5}}{\sqrt[4]{3}} \text{ as required}$$

$$h = \frac{\sqrt{3}}{2}b = 2\sqrt{5}\sqrt[4]{3} \text{ cm} \quad [1A]$$

$$l = \frac{240 - \sqrt{3}b^2}{6b} \text{ cm}$$

$$= \frac{(240 - 80)\sqrt[4]{3}}{24\sqrt{5}} \quad [1M]$$

$$= \frac{20 \times \sqrt[4]{3}}{3\sqrt{5}}$$

$$= \frac{4\sqrt{5} \times \sqrt[4]{3}}{3}$$

$$= \frac{4\sqrt{5}}{3^{\frac{3}{4}}} \text{ cm} \quad [1A]$$

$$\text{e. } V = \frac{b(80\sqrt{3} - b^2)}{8} \quad [1M]$$

$$= \frac{4\sqrt{5}}{\sqrt[4]{3}} \times \frac{80\sqrt{3} - \frac{80\sqrt{3}}{3}}{8}$$

$$= \frac{4\sqrt{5}}{\sqrt[4]{3}} \times \frac{20\sqrt{3}}{3}$$

$$= \frac{80\sqrt{5}}{3^{\frac{3}{4}}} \text{ cm}^3 \quad [1A]$$

Question 2

a. $N = N_0 e^{-m t}$

$$140 = 200 e^{-m \times 1}$$

$$m = -\log_e \left(\frac{140}{200} \right)$$

$$m = \log_e \left(\frac{10}{7} \right)$$

[1M]

b. $N = N_0 e^{-m t}$

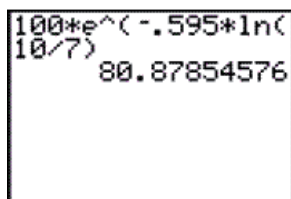
$$\text{percentage} = \frac{200 e^{-\log_e \left(\frac{10}{7} \right) \times 0.595}}{200} \times \frac{100}{1}$$

$$= 100 \times e^{-\log_e \left(\frac{10}{7} \right) \times 0.595}$$

[1M]

$$= 81\%$$

[1A]



Handwritten calculator work showing the calculation of $100 * e^{(-.595 * \ln(10/7))}$ resulting in 80.87854576.

c. $80 = 100 e^{-m \times 0.5}$

$$m = -\frac{1}{0.5} \log_e \left(\frac{80}{100} \right)$$

$$m = 2 \log_e \left(\frac{5}{4} \right)$$

$$25 = 100 e^{-2 \log_e \left(\frac{5}{4} \right) \times t}$$

$$t = \frac{-\log_e (0.25)}{2 \log_e \left(\frac{5}{4} \right)}$$

[1M]

$$t = 3.106$$

It will take 3106 years for 75% replacement.

[1A]

- d. i. At the initial meeting, $t = 0$.

$$B(0) = \frac{60}{1 + 3e^0}$$

$$= 15$$

15 “jargon” words were used initially.

[1A]

d. ii $40 = \frac{60}{1 + 3e^{-0.05t}}$

Solving for t gives the result $t \approx 35.84$

[1M]

The buzz word count achieved at 36 months.

[1A]

OR

Alternatively

$$40 = \frac{60}{1 + 3e^{-0.05t}}$$

$$3e^{-0.05t} = \frac{1}{2}$$

$$t = -\frac{1}{0.05} \log_e \left(\frac{1}{6} \right)$$

$$t \approx 35.84$$

The jargon word count achieved at 36 months

e. For a probability density function

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$0 + A \int_0^1 (t-1)^2 dt = 1$$

[1M]

$$A \times \frac{1}{3} = 1$$

[1M]

f. $\mu = \int_{-\infty}^{\infty} x f(x) dx$ for a probability density function

$$\mu = \int_0^1 t(t-1)^2 dt$$

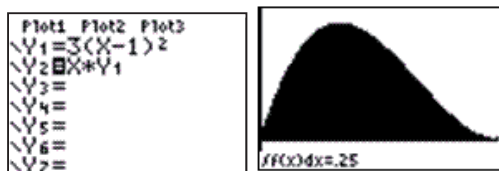
$$= 0.25$$

[1M]

The mean is 0.25 thousand years = 250 years, as required.

[1A]

OR



g. $\Pr(T > 200) = 0.003 \int_2^{10} (t-10)^2 dt$

[1M]

$$\Pr(T > 200) = 0.512$$

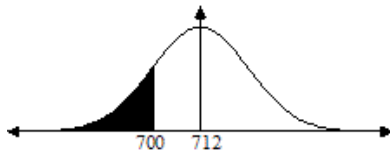
[1A]

Question 3

a. $X \sim N(\mu = 712, \sigma^2 = 20^2)$

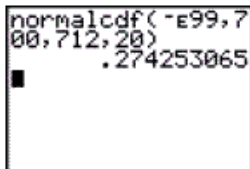
$$\Pr(X < 700) = 0.274$$

[1M]

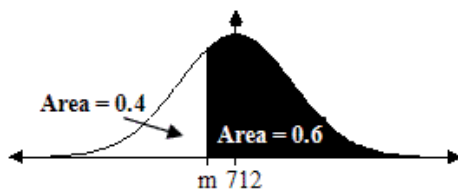


The proportion of cartons less than 700 g is 0.274.

[1A]



b. $X \sim N(\mu = 712, \sigma^2 = 20^2)$



$$\Pr(X > m) = 0.6$$

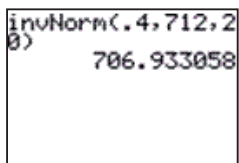
$$\Pr(X \leq m) = 0.4$$

[1M]

$$m = 706.9$$

60% of cartons exceed 706.9 grams.

[1A]



c. $\Pr(X < 695 | X < 700)$

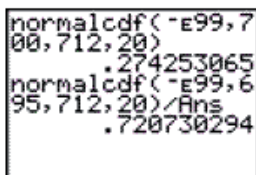
$$= \frac{\Pr(X < 695 \cup X < 700)}{\Pr(X < 700)}$$

$$= \frac{\Pr(X < 695)}{\Pr(X < 700)}$$

[1M]

$$\Pr(X < 695 | X < 700) = 0.721$$

[1A]



d. Let Y be the number of underweight cartons.

$$Y \sim Bi(8, 0.274)$$

(The probability of a carton being underweight was found in part a. to be 0.274)

$$\Pr(Y \geq 3) = 1 - \Pr(Y \leq 2)$$

[1M]

$$\Pr(Y \geq 3) = 0.383$$

[1A]

```
normalcdf(-E99,7
00,712,20)
.274253065
1-binomcdf(8,Ans
,2)
.382636634
```

e.

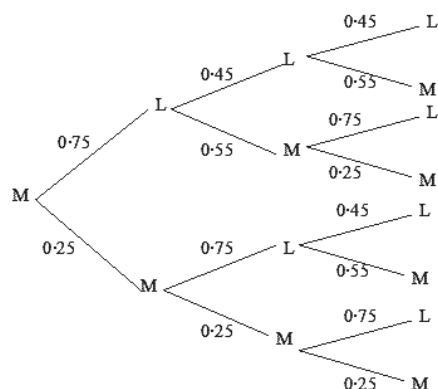
	Large today	Medium today
Large Tomorrow	0.45	0.75
Medium Tomorrow	0.55	0.25

i. $\Pr(L,L,M) = 0.75 \times 0.45 \times 0.55$

$$\Pr(L,L,M) = 0.1856$$

[1A]

ii. TUE WED THU FRI



[1M]

$$0.75 \times (0.45)^2 + (0.75)^2 \times 0.55 + 0.25 \times 0.75 \times 0.45 + (0.25)^2 \times 0.75$$

[1M]

$$= 0.5925$$

[1A]

```
.75*.45^2+.75^2*.5
5+.25*.75*.45+.2
5^2*.75
.5925
```

Question 4

a. $\sin(2x) = \frac{1}{2}$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

[1M]

$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$

[1A]

b. i. A dilation of a factor of 3 from the x-axis

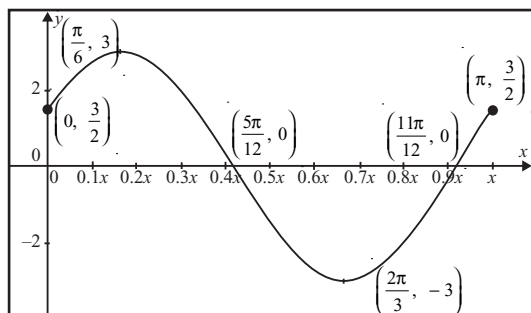
[1A]

and a translation of $\frac{\pi}{12}$ units to the left.

[1A]

(The order does not matter.)

ii.



Correct shape

[1A]

Correct coordinates for the x-intercepts

$$\left(\frac{5\pi}{12}, 0\right) \text{ and } \left(\frac{11\pi}{12}, 0\right)$$

[1A]

Correct end points with closed circles

$$\left(0, \frac{3}{2}\right) \text{ and } \left(\pi, \frac{3}{2}\right)$$

[1A]

Correct turning points

$$\left(\frac{\pi}{6}, 3\right) \text{ and } \left(\frac{2\pi}{3}, -3\right)$$

[1A]

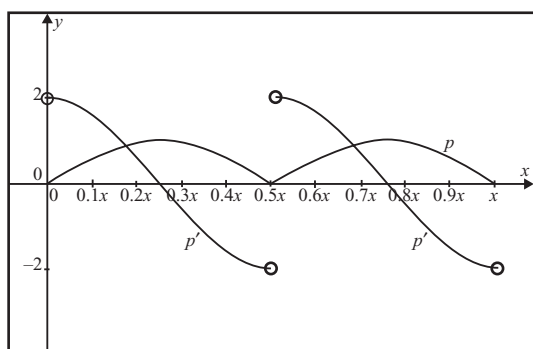
$$\text{c. } \frac{d}{dx} \sin(2x) = 2 \cos(2x) \quad [1A]$$

$$\begin{aligned} & \frac{d}{dx} 2 \sin(x) \cos(x) \\ &= 2(\cos^2(x) - \sin^2(x)) \\ &= 2(\cos^2(x) - (1 - \cos^2(x))) \\ &= 4 \cos^2(x) - 2 \end{aligned} \quad [1A]$$

Hence $2 \cos(2x) = 4 \cos^2(x) - 2$ as required

$$\text{d. i. } p'(x) = \begin{cases} 2 \cos(2x) & \text{if } \sin(2x) > 0 \\ -2 \cos(2x) & \text{if } \sin(2x) < 0 \end{cases} \quad [2A]$$

ii.



Correct graph for p [1A]

Correct graph for p' [1A]

$$\text{x-intercepts of } p': \frac{\pi}{4}, \frac{3\pi}{4}$$

Open circles

[1A]

$$\text{iii. } \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin(2x) - 2 \cos(2x)) dx \quad [1M]$$

$$= 1.5 \text{ unit}^2$$

[1A]

