



The Mathematical Association of Victoria MATHEMATICAL METHODS

Trial written examination 2

2008

Reading time: 15 minutes

Writing time: 2 hours

Student's Name:

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

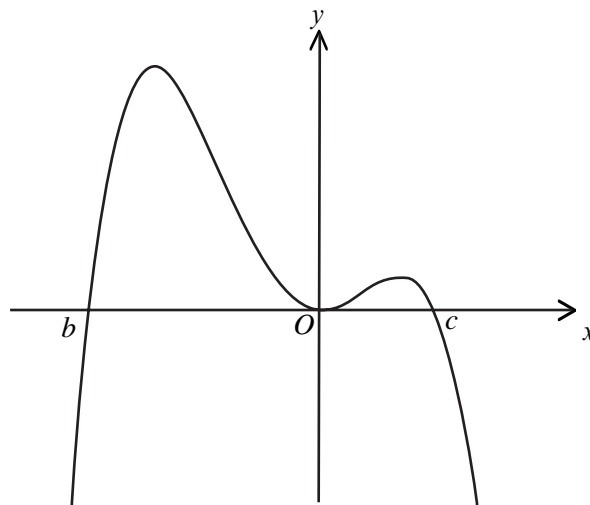
No marks will be given if more than one answer is completed for any question.

Question 1

The equation of the cubic function for which the graph passes through the points $(0, 1)$, $(1, \frac{4}{3})$, $(-1, -\frac{4}{3})$ and $(2, \frac{5}{3})$ is

- A. $y = 0.33x^3 - x^2 + x + 1$
- B. $y = 0.3x^3 - x^2 + x + 1$
- C. $y = x^3 - 3x^2 + 3x + 3$
- D. $3y = x^3 - 3x^2 + 3x + 3$
- E. $y = 0.33x^3 + x^2 + x + 1$

Question 2



Part of the graph of $y = h(x)$ is shown above. If a , b and c are real constants and $a > 0$ which one of the following could be the rule for h ?

- A. $h(x) = ax(x-b)(x-c)$
- B. $h(x) = -ax(x+b)(x-c)$
- C. $h(x) = ax^2(x+b)(x-c)$
- D. $h(x) = -ax^2(x+b)(x-c)$
- E. $h(x) = -ax^2(x-b)(x-c)$

Question 3

If $f(x) = e^{3x}$ and $g(x) = \log_e |x|$ then the rule, domain and range of $f(g(x))$ are respectively

- A. $f(g(x)) = |x^3|, R \setminus \{0\}, R^+$
- B. $f(g(x)) = |x^3|, R \setminus \{0\}, R^+ \cup \{0\}$
- C. $f(g(x)) = 3x, R, R$
- D. $f(g(x)) = |x^3|, R^+, R^+$
- E. $f(g(x)) = |x^3|, R^+, R \setminus \{0\}$

Question 4

If $f: (-\infty, 3] \rightarrow R$ where $f(x) = (x-3)^{\frac{2}{3}} + 1$ then f^{-1} is

- A. $f^{-1}: R \rightarrow R$, where $f^{-1}(x) = (x-1)^{\frac{3}{2}} + 3$
- B. $f^{-1}: [1, \infty) \rightarrow R$, where $f^{-1}(x) = (x-1)^{\frac{3}{2}} + 3$
- C. $f^{-1}: [1, \infty) \rightarrow R$, where $f^{-1}(x) = 3 - (x-1)^{\frac{3}{2}}$
- D. $f^{-1}: (-\infty, 3] \rightarrow R$, where $f^{-1}(x) = 3 - (x-1)^{\frac{3}{2}}$
- E. $f^{-1}: R \setminus \{3\} \rightarrow R$, where $f^{-1}(x) = \frac{2}{3(x-3)^{\frac{1}{3}}}$

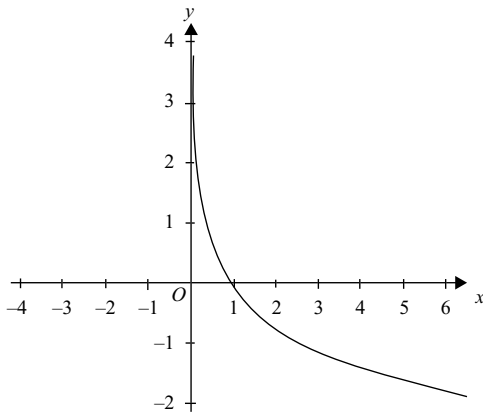
Question 5

The graphs of $y = x - 8$ and $y = 2x^2 + mx$, where m is a real constant, intersect at exactly one point for

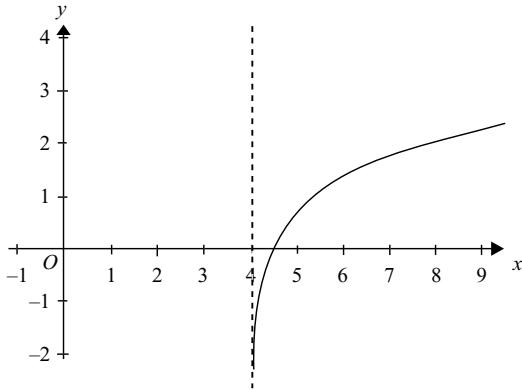
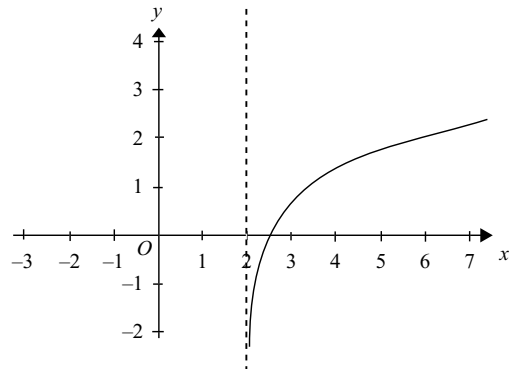
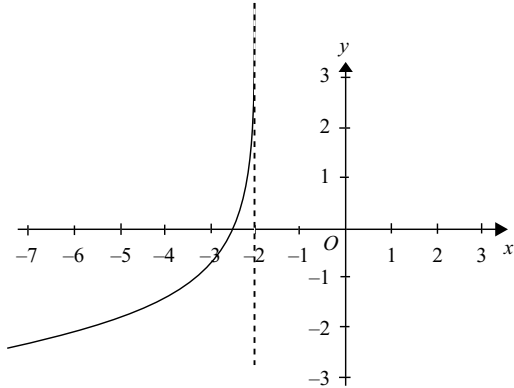
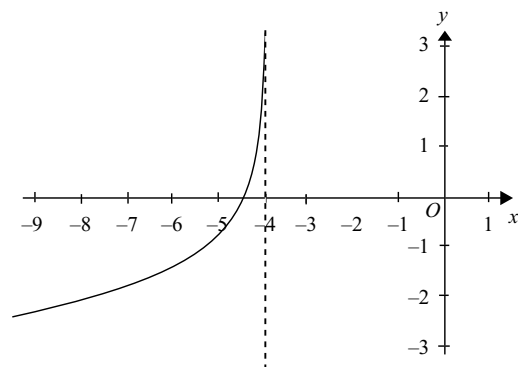
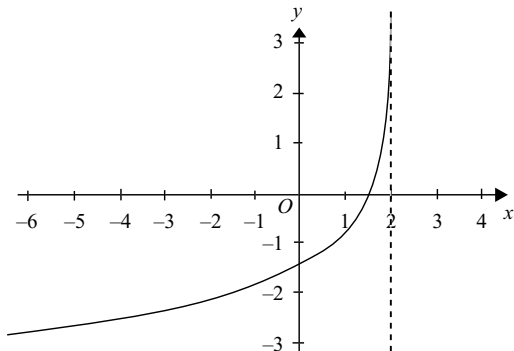
- A. $m = 9$ only
- B. $m = 7$ only
- C. $m = -7$ or $m = 9$
- D. $m = -7$ or $m = 7$
- E. $m = -9$ or $m = 9$

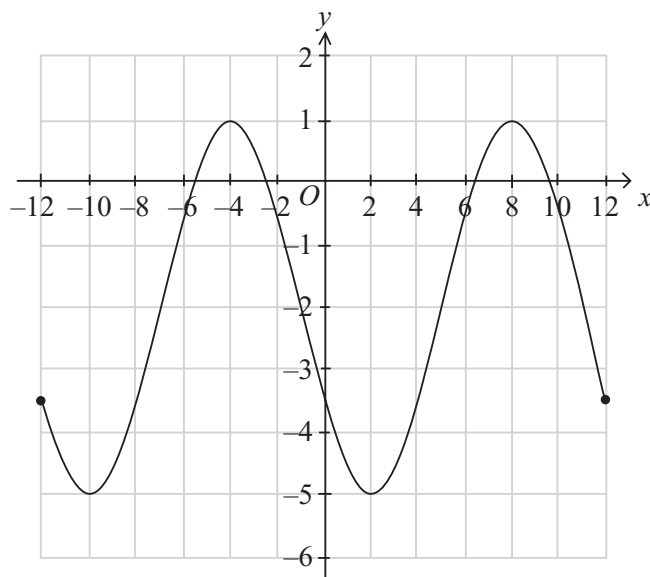
Question 6

The graph of $y = h(x)$ is shown below.



The graph of $y = -h(2x - 4)$ could be

A.**B.****C.****D.****E.**

Question 7

The diagram above shows the graph of $y = f(x)$. Which one of the following could be the rule for f ?

- A. $f(x) = 6 \sin\left(\frac{\pi}{6}(x-5)\right) - 2$
- B. $f(x) = 3 \sin\left(\frac{\pi}{12}(x-5)\right) - 2$
- C. $f(x) = 3 \sin\left(\frac{\pi}{6}(x+5)\right) - 2$
- D. $f(x) = 3 \sin\left(\frac{\pi}{12}(x+2)\right) - 5$
- E. $f(x) = 3 \sin\left(\frac{\pi}{6}(x-5)\right) - 2$

Question 8

The graph of $f: [-\pi, \pi] \rightarrow \mathbb{R}$ where $f(x) = -4\tan(3x)$ has

- A. no asymptotes
- B. 2 asymptotes
- C. 4 asymptotes
- D. 6 asymptotes
- E. 8 asymptotes

Question 9

The sequence of transformations that could be used to obtain the graph of $y = e^{-2(x+1)}$ from the graph of $y = e^x$ is

- A. a reflection in y -axis, a dilation by a factor of $\frac{1}{2}$ from the y -axis and a translation 1 unit left
- B. a reflection in x -axis, a dilation by a factor of 2 from the y -axis and a translation 1 unit left
- C. a reflection in y -axis, a dilation by a factor of $\frac{1}{2}$ from the y -axis and a translation 1 unit right
- D. a reflection in x -axis, dilation by a factor of 2 from the x -axis and a translation 1 unit right
- E. a reflection in y -axis, dilation by a factor of $\frac{1}{2}$ from the x -axis and a translation 1 unit right

Question 10

$e^{2x} + be^x + 1 = 0$, where b is real constant, will have no real solutions if

- A. $\{b : -2 \leq b < 2\}$
- B. $\{b : -2 < b \leq 2\}$
- C. $\{b : b < -2\} \cup \{b : b > 2\}$
- D. $\{b : b \leq -2\} \cup \{b : b \geq 2\}$
- E. $\{b : -2 < b < 2\}$ only

Question 11

If $f(x) = \begin{cases} x^2 + 4x + a & \text{for } x \geq 0 \\ bx + c & \text{for } x < 0 \end{cases}$, where a , b and c are real constants, then $f(x)$ will be differentiable at

$x = 0$ if

- A. $a = c$ and $b \in \mathbb{R}$
- B. $a = c$ and $b = 4$ only
- C. $b = 4$ and $a, c \in \mathbb{R}$
- D. $b = 2$ and $c = 4$
- E. $a = b = c$

Question 12

Using the approximation formula, $f(x + h) \approx hf'(x) + f(x)$ then the change in f , where $f(x) = \log_e(2x + 1)$, as x decreases from 1 to 0.9 is

- A. $-\frac{1}{15}$
- B. $\frac{1}{15}$
- C. $\log_e\left(\frac{14}{15}\right)$
- D. $-\frac{1}{15} + \log_e(3)$
- E. $\frac{1}{15} + \log_e(3)$

Question 13

The equation of the normal to the curve with equation $y = x^5 + 2x$ at $x = 0$ is

- A. $x = 0$
- B. $y = 0$
- C. $y = 2x$
- D. $y = -\frac{1}{2}x$
- E. $y = 2$

Question 14

If $\frac{x^2}{y} = 1$ and $p = 5x + 3y$ then p will have

- A. a maximum value when $x = -\frac{5}{3}$
- B. a minimum value when $x = -\frac{5}{3}$
- C. a maximum value when $x = \frac{25}{36}$
- D. a maximum value when $x = -\frac{5}{6}$
- E. a minimum value when $x = -\frac{5}{6}$

Question 15

If a particle is moving in a straight line such that its position x cm from a point O at time t seconds is given by $x = \sqrt{1-t}$ for $0 < t < 1$ then its acceleration, a cms⁻², where acceleration is the rate of change of velocity, is given by

- A. $a = -\frac{1}{2\sqrt{1-t}}$
- B. $a = -\frac{1}{4(1-t)^{\frac{3}{2}}}$
- C. $a = \frac{1}{4(1-t)^{\frac{3}{2}}}$
- D. $a = \frac{1}{2\sqrt{1-t}}$
- E. $a = -\frac{2}{3}(1-t)^{\frac{3}{2}}$

Question 16

$\int_0^1 \left(\frac{e^{2x^2}}{1+x} \right) dx$ correct to one decimal place is

- A. -1.4
- B. -1.5
- C. 2.9
- D. 1.4
- E. 1.5

Question 17

The area bounded by the curves $y = \cos(2x)$ and $y = \sin(x)$ for $0 \leq x \leq \pi$ can be found by evaluating

- A. $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin(x) - \cos(2x)) dx$
- B. $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\cos(2x) - \sin(x)) dx$
- C. $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (\sin(x) - \cos(2x)) dx$
- D. $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (\cos(2x) - \sin(x)) dx$
- E. $2 \int_0^{\frac{5\pi}{6}} (\cos(2x) - \sin(x)) dx + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin(x) - \cos(2x)) dx$

Question 18

The probability distribution for a continuous random variable, T , is defined by the probability density

$$\text{function } f(t) = \begin{cases} k & -\pi \leq t \leq \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

If k is a real constant, the value of k is

- A. $\frac{2}{3}$
- B. $\frac{3\pi}{2}$
- C. $\frac{2}{3\pi}$
- D. $\frac{2\pi}{3}$
- E. $\sin(t)$

Question 19

Suppose that in a particular year the percentage increase in the value of stocks listed on the Australian Stock Exchange was a normally distributed random variable with a mean of 5% and standard deviation of 10%.

The proportion of stocks that would have **decreased** in value in that particular year is closest to

- A. 0.31
- B. 0.5
- C. 0.69
- D. 0.05
- E. 0.1

Question 20

In the telephone directory there are eight people with the same initial and surname as that of my friend, Stewart. I randomly select and dial telephone numbers from the eight listed. If I dial a number only once, the probability that the third number dialled is Stewart's number is given by

- A. $\frac{7}{8} \times \frac{6}{8} \times \frac{1}{8}$
- B. $\frac{7}{8} \times \frac{6}{7} \times \frac{1}{6}$
- C. $\frac{1}{8} \times \frac{1}{7} \times \frac{5}{6}$
- D. $\left(\frac{7}{8}\right)^2 \times \frac{1}{8}$
- E. $\left(\frac{7}{8}\right)^3$

Question 21

The probability distribution of a discrete random variable, X , is shown below.

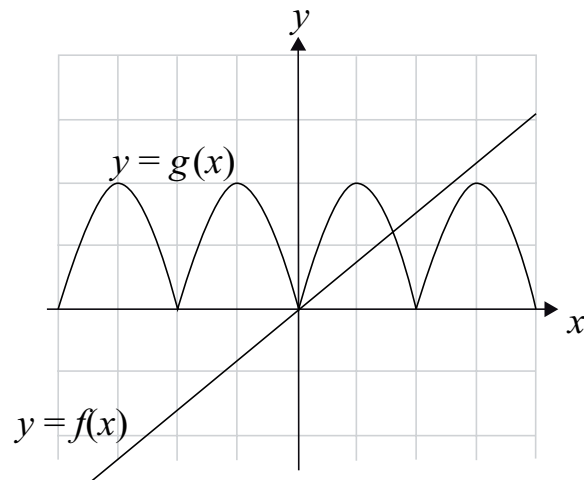
x	0	1	$\sqrt{2}$
$\Pr(X=x)$	h	h^2	$\frac{1}{2} - h$

If h is a real constant, the mean of X is

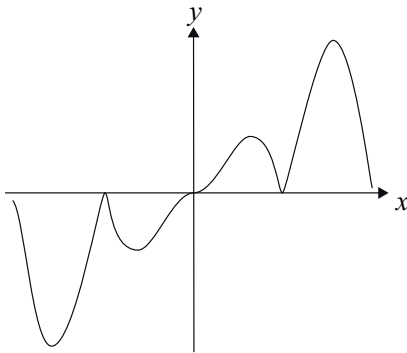
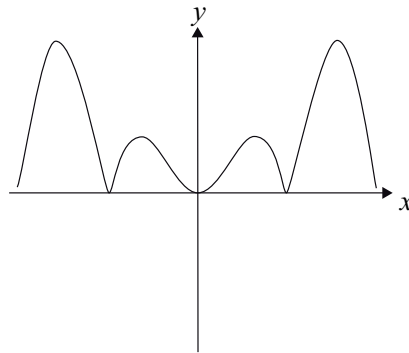
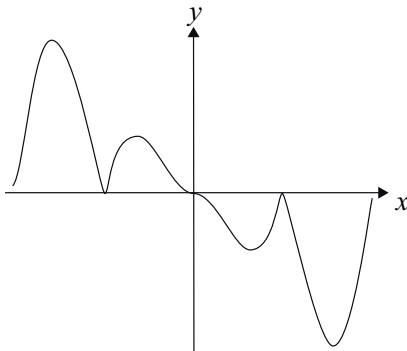
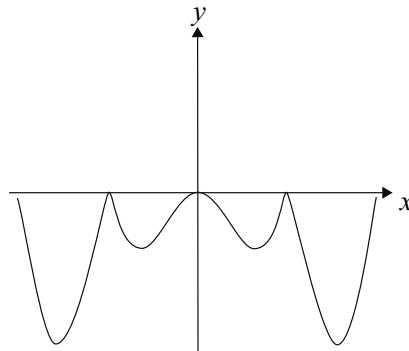
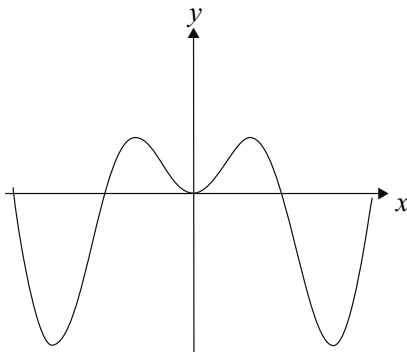
- A. $\frac{\sqrt{2}}{2}$
- B. $\frac{\sqrt{2}-1}{2}$
- C. $\frac{1-\sqrt{2}}{2}$
- D. $\frac{\sqrt{2}}{2} - 1$
- E. $\frac{2}{\sqrt{2}}$

Question 22

The graphs of $y = f(x)$ and $y = g(x)$ are shown.



The graph of $y = (fg)(x)$ is best represented by

A.**B.****C.****D.****E.**

Working space

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

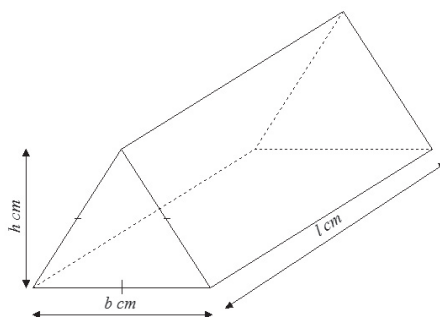
In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A chocolate manufacturer wants to promote a particular brand of chocolate called Toby by maximising the amount of chocolate in their new Toby bar. The Toby bar is in the shape of a triangular prism as shown below, where the triangle has base, b cm and height, h cm and the length of the bar is l cm.



The surface area of the Toby bar is 120 cm^2 and the triangle is an **equilateral** triangle, where $3 \leq b \leq 9$.

a. Show that $l = \frac{240 - \sqrt{3}b^2}{6b}$.

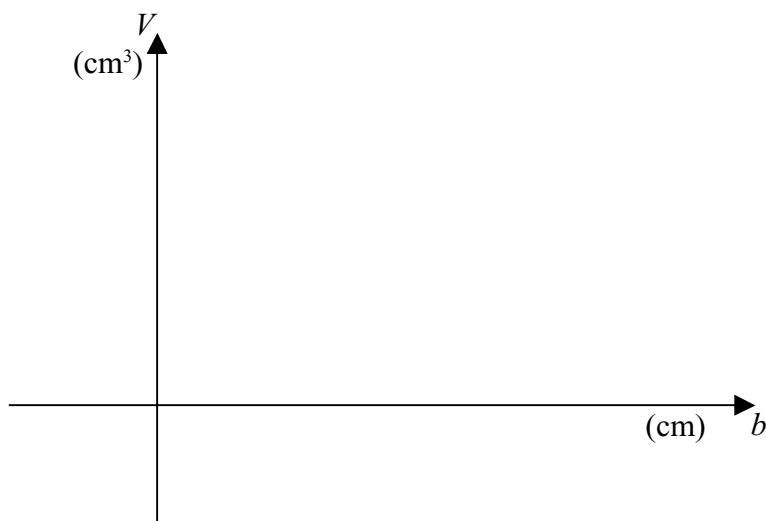
3 marks

- b. Hence, show that the volume, $V \text{ cm}^3$, of the chocolate bar in terms of b is given by

$$V = \frac{b(80\sqrt{3} - b^2)}{8}.$$

2 marks

- c. Sketch the graph of V against b on the set of axes below. Give the coordinates of the end points and turning point correct to two decimal places.



3 marks

- Hence find the other dimensions (h and l) of the chocolate that will give maximum volume.

[illegible]

e. What is the maximum amount of chocolate, in cm^3 , the manufacturer can put in the bar? Give your answer in the form $\frac{C\sqrt{D}}{E^F}$, where C, D, E and F are real constants.

Total 15 marks

Question 2

Glottochronology is the study of the age of languages.

The number of ancient words retained in a language can be modelled by the function $N = N_0 e^{-mt}$, $t \geq 0$, where N is the number of ancient words retained after t **thousand** years, N_0 is the initial number of ancient words and m is a positive real constant.

- a. In 2005, Graham, a glottochronology scholar, studied a list of 200 words from an ancient Chinese text dated 1005 AD. He found that 140 words were still used, 1000 years later, in modern Mandarin

Chinese. According to these results, show that $m = \log_e \left(\frac{10}{7} \right)$.

1 mark

- b. According to this model, what **percentage** of ancient words, from the 1005 AD text, were still in use in 1600 AD? Give your answer correct to the nearest integer.

2 marks

Graham also studied the divergence of the Italian language from Classical Latin. Again, he used the function $N = N_0 e^{-mt}$, $t \geq 0$, to model the new situation.

- c. Graham found that 80% of words in a Classical Latin manuscript dated 200 AD were retained in the language of the Tuscany region of Italy in 700 AD. At this rate of word replacement, how many years will it take for 75% of words from Classical Latin to be replaced in Tuscan Italian? Answer correct to the nearest year.

2 marks

As a diversion during tedious staff meetings, Graham kept count of the number of times that a “jargon” word was used by his new boss, John. He found that the number of times a “jargon” word was used in a meeting, B , t months after John’s appointment as boss, could be modelled by the function

$$B(t) = \frac{60}{1 + 3e^{-0.05t}}, \quad t \geq 0.$$

- d. According to this model

- i. how many times did John use a “jargon” word at the initial staff meeting?

- ii. how many months after his appointment will John use a “jargon” word 40 times in a meeting? (Answer correct to the nearest integer)

(1 + 2 = 3 marks)

Graham also studied the divergence of the English language from German. He found that the life of an old Germanic word in English (that is, the time taken for an old Germanic word to be replaced), is a random variable, T **thousand** years, with the probability density function shown below, where A is a real constant.

$$f(t) = \begin{cases} A(t-1)^2 & , 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- e. Show that $A = 3$.

2 marks

- f. Show that, according to this model, the expected or mean life of an old Germanic word in the English language is 250 years.

2 marks

- g. According to this model, what proportion of old Germanic words, have a life greater than 200 years?

2 marks

Total 14 Marks

Working space

Question 3

The *Yolks On You* company markets its large-sized eggs in cartons labelled 700 g. However, quality controllers have found that the total mass of eggs in each of the cartons, X grams, is a normally distributed random variable with a mean of 712 grams and a standard deviation of 20 grams.

- a. In what proportion of cartons, correct to three decimal places, do the eggs have a total mass of less than 700grams?

2 marks

- b. 60% of cartons contain eggs with a total mass greater than m . What is the value of m , in grams, correct to one decimal place?

2 marks

Cartons that are found to contain less than 700 grams of eggs are classified as underweight and are relabelled as medium-sized.

- c. What is the probability that a carton of eggs that is relabelled as medium-sized contain less than 695 grams of eggs? Answer correct to three decimal places.

2 marks

From the thousands of cartons that are labelled as large-sized on a particular day, a quality controller weighs the contents of 8 cartons in turn.

- d. What is the probability, correct to three decimal places, that at least three cartons will be classified as underweight?

2 marks

Celebrity chef, Jamie Oliveloaf, buys his eggs daily from *Yolks On You* for his restaurant in Thyme Square.

Each day Jamie orders medium-sized or large-sized eggs. If he orders medium-sized one day, the probability that he orders large-sized the following day is 0.75. However, if he orders large-sized one day, the probability that he orders medium-sized the following day is 0.55.

- e. Jamie orders medium-sized eggs on Tuesday.

- i. What is the probability, correct to four decimal places, that he will order large-sized eggs on Wednesday and Thursday and medium-sized eggs on Friday of that week?

- ii. What is the probability that he will order large-sized eggs on Friday of that week?

1 + 3 = 4 marks

Total 12 marks

SECTION 2 – continued
TURN OVER

Question 4

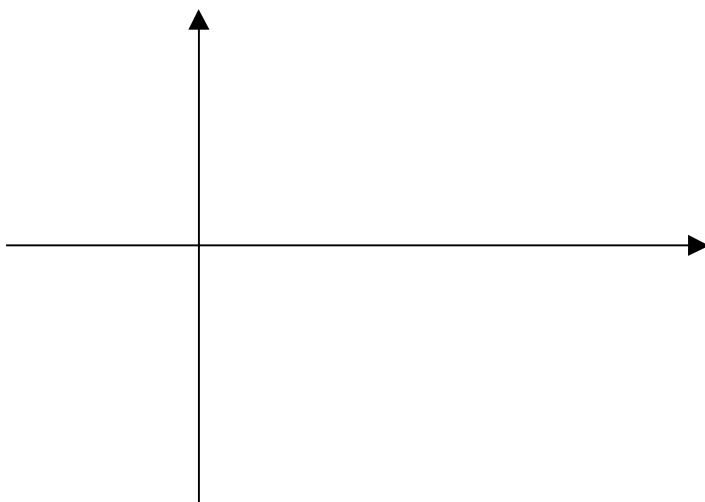
- a. Find the $\{x : \sin(2x) = \frac{1}{2}\}$ for $x \in [0, \pi]$.

2 marks

- b. i. State the transformations of the graph of $g(x) = \sin(2x)$ to get the graph of $h(x) = 3\sin\left(2\left(x + \frac{\pi}{12}\right)\right)$.

2 marks

- ii. Sketch the graph of $f: [0, \pi] \rightarrow \mathbb{R}$, where $f(x) = 3\sin\left(2\left(x + \frac{\pi}{12}\right)\right)$ on the set of axes below, clearly labelling intercepts and turning points with their coordinates.



4 marks

- c. By differentiating both sides of the equation $\sin(2x) = 2\cos(x)\sin(x)$, show that $2\cos(2x) = 4\cos^2(x) - 2$.

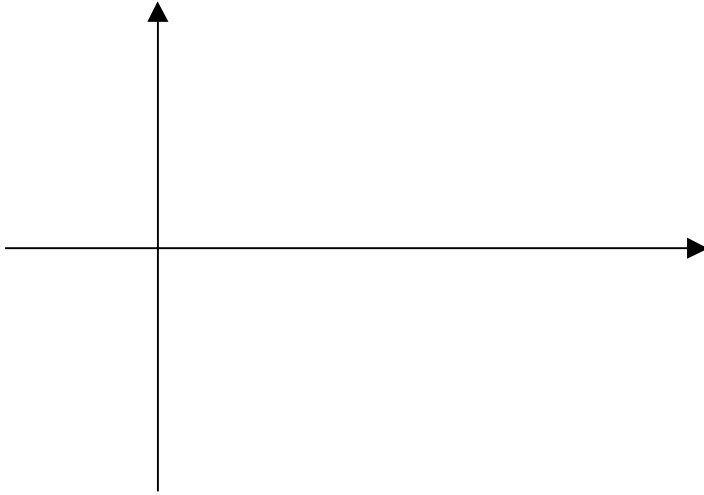
2 marks

- d. i. Write down the derivative of $p(x) = |\sin(2x)|$ as a hybrid function by putting the appropriate expressions on the dotted lines.

$$p'(x) = \begin{cases} \dots\dots\dots & \text{if } \sin(2x) > 0 \\ \dots\dots\dots & \text{if } \sin(2x) < 0 \end{cases}$$

2 marks

- ii. Sketch the graphs of p and p' for $x \in (0, \pi)$ on the set of axes below.



3 marks

- iii. Find the area bounded by p, p' and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$.

2 marks

Total 17 marks

MULTIPLE CHOICE ANSWER SHEET

Student Name:

Circle the letter that corresponds to each correct answer

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

MATHEMATICAL METHODS

Trial written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time
This formula sheet is provided for your reference

Mathematical Methods and Mathematical Methods CAS

Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean: $\mu = E(X)$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$