



INSIGHT

YEAR 12 Trial Exam Paper

2012

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 2 hours

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT have to be cleared.
- Students are NOT permitted to bring sheets of paper or white-out liquid/tape into the examination.

Materials provided

- The question and answer book of 27 pages, a formula sheet, and an answer sheet for the multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The ellipse with equation $9x^2 + 16y^2 - 18x + 64y - 71 = 0$ has

- A. centre $(1, -2)$ with major axis of length 4 units.
- B. centre $(-1, 2)$ with major axis of length 8 units.
- C. centre $(1, -2)$ with minor axis of length 8 units.
- D. centre $(1, -2)$ with major axis of length 8 units.
- E. centre $(-1, 2)$ with major axis of length 16 units.

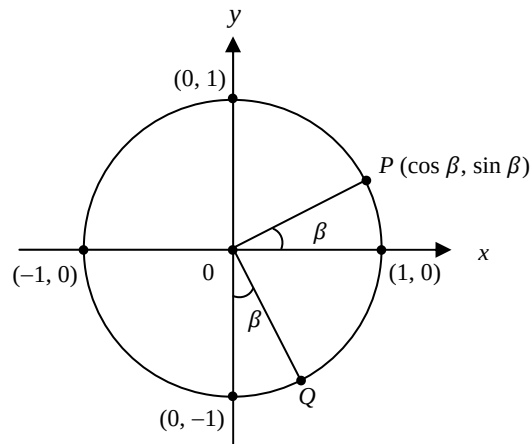
Question 2

The curve $y = \left(x - 2 + \frac{1}{\sqrt{x-2}} \right) \left(x - 2 - \frac{1}{\sqrt{x-2}} \right) + 4$ has a

- A. curved asymptote $y = x^2 - 4x - 8$ and vertical asymptote $x = 2$.
- B. curved asymptote $y = (x - 2)^2$ and vertical asymptote $x = 2$.
- C. curved asymptote $y = x^2 - 4x + 8$ and vertical asymptote $x = 2$.
- D. curved asymptote $y = x^2 - 4x + 8$ and vertical asymptote $x = -2$.
- E. horizontal asymptote $y = 4$ and vertical asymptote $x = 2$.

Question 3

In the diagram below, P has coordinates $(\cos \beta, \sin \beta)$.
Hence, Q would be the point with coordinates



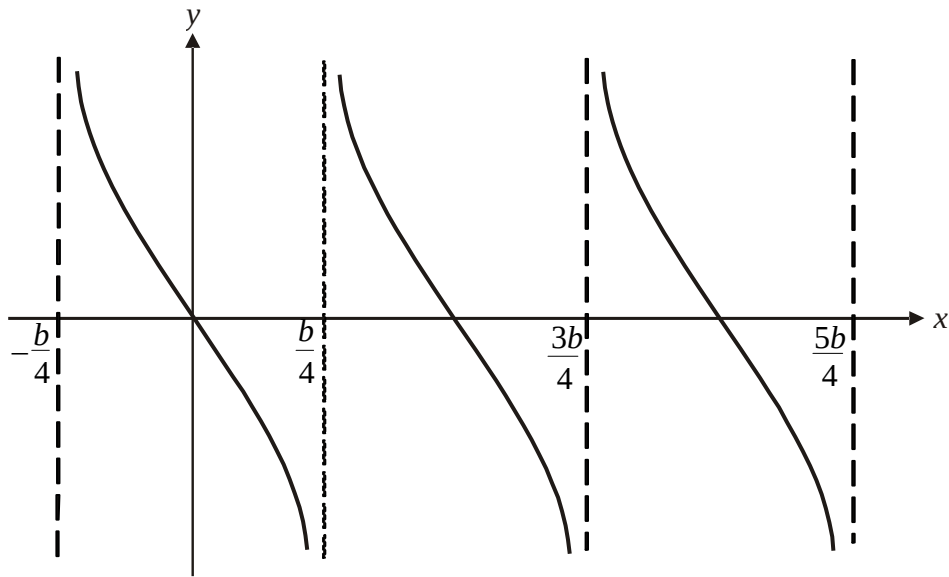
- A. $(\sin \beta, -\cos \beta)$
- B. $(\cos \beta, \sin \beta)$
- C. $(\sin \beta, \cos \beta)$
- D. $(\cos \beta, -\sin \beta)$
- E. $(-\sin \beta, -\cos \beta)$

Question 4

If $\sin(x) = -\frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$ then $\sec(x)$ equals

- A. $-\frac{4}{3}$
- B. $-\frac{4\sqrt{7}}{7}$
- C. $-\frac{\sqrt{7}}{4}$
- D. $\frac{4}{\sqrt{7}}$
- E. $\frac{3\sqrt{7}}{7}$

Question 5



The function whose graph is shown above, where $b > 0$, could have the rule given by

- A. $y = -\tan\left(\frac{2bx}{\pi}\right)$
- B. $y = \cot\left(\frac{2\pi x}{b}\right)$
- C. $y = -\tan\left(\frac{\pi x}{b}\right)$
- D. $y = \cot\left[\frac{2\pi}{b}\left(x - \frac{b}{4}\right)\right]$
- E. $y = \cot\left[\frac{2\pi}{b}\left(x - \frac{b}{2}\right)\right]$

Question 6

For $y = k - a \sin^{-1}(2x - b)$, $a > 0$, $b > 0$, the maximal domain and range are

- A. domain $\left[\frac{-b}{2}, \frac{b}{2}\right]$, range $\left[\frac{a - k\pi}{2}, \frac{a + k\pi}{2}\right]$
- B. domain $\left[\frac{b-1}{2}, \frac{b+1}{2}\right]$, range $\left[\frac{a - k\pi}{2}, \frac{a + k\pi}{2}\right]$
- C. domain $\left[\frac{b-1}{2}, \frac{b+1}{2}\right]$, range $\left[\frac{2k - a\pi}{2}, \frac{2k + a\pi}{2}\right]$
- D. domain $\left[\frac{-b}{2}, \frac{b}{2}\right]$, range $\left[\frac{k - 2\pi}{2}, \frac{k + 2\pi}{2}\right]$
- E. domain $\left[\frac{-b}{2} - 1, \frac{b}{2} + 1\right]$, range $\left[k - \frac{a\pi}{2}, k + \frac{a\pi}{2}\right]$

Question 7

If $z = 1 + i$, then $\text{Arg}(i^3 z)$ is

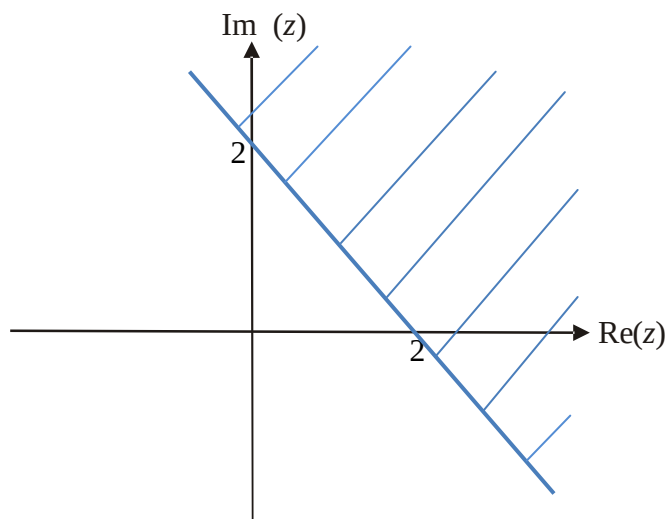
- A. $\frac{7\pi}{4}$
- B. $\frac{5\pi}{4}$
- C. $-\frac{3\pi}{4}$
- D. $\frac{3\pi}{4}$
- E. $-\frac{\pi}{4}$

Question 8

The complex relation $\text{Re}(z - 1) \times \text{Im}(z - 1) = 1$ can be represented on a Cartesian plane as a

- A. hyperbola with asymptotes $x = 1$, $y = 0$.
- B. hyperbola with asymptotes $y = \pm x$.
- C. hyperbola with asymptotes $x = 0$, $y = 1$.
- D. straight line with equation $y = 1 - x$.
- E. hyperbola with equation $y = \frac{1}{x-1} + 1$.

Question 9



The shaded region on the Argand diagram above represents

- A. $|z + 2 + 2i| \geq |z|$
- B. $|z - 2 - 2i| \leq |z - 2|$
- C. $|z - 2| \geq |z - 2i|$
- D. $|z - 2i| \leq |z - 2|$
- E. $|z| \geq |z - 2 - 2i|$

Question 10

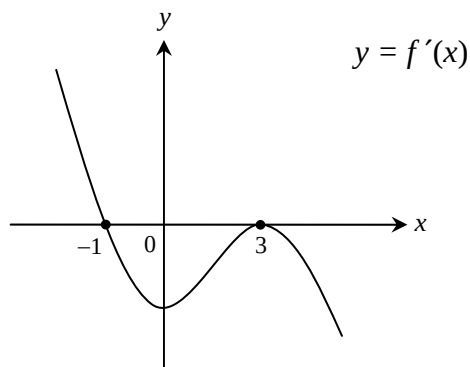
Given that $z + i$ is a factor of $P(z) = z^3 - z^2 + z - 1$, which one of the following statements is **not** true?

- A. $P(z) = 0$ has three solutions.
- B. $P(-i) = 0$.
- C. $P(z) = 0$ has two real solutions.
- D. $P(z)$ can be factorised over C .
- E. $P(i) = 0$.

Question 11

If $\frac{da}{dy} = 9 + a^2$ and $y = 0$ when $a = 0$, then y is equal to

- A. $\tan^{-1}\left(\frac{a}{3}\right)$
- B. $\frac{1}{3}\tan^{-1}(3a)$
- C. $3\tan^{-1}a$
- D. $\frac{1}{3}\tan^{-1}\left(\frac{a}{3}\right)$
- E. $\frac{1}{3}\tan\left(\frac{a}{3}\right)$

Question 12

If $f(x)$ is an antiderivative of $f'(x)$, the graph of $y = f(x)$ has

- A. a stationary point of inflexion at $x = 0$ and a local maximum at $x = -1$
- B. stationary points of inflexion at $x = 0$ and $x = 3$ and a local maximum at $x = -1$
- C. a stationary point of inflexion at $x = 3$ and a local maximum at $x = -1$
- D. a local minimum at $x = 0$ and a local maximum at $x = 3$
- E. a stationary point of inflexion at $x = 3$ and a local minimum at $x = -1$

Question 13

For the curve $xy^3 - 2x^2 = 7$

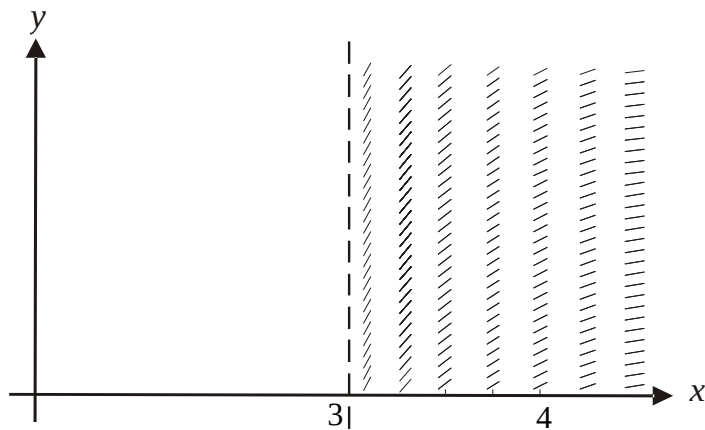
- A. $\frac{dy}{dx} = \frac{3x - 4y^3}{3xy^2}$
- B. $\frac{dy}{dx} = \frac{7 + 4x}{6x}$
- C. $\frac{dy}{dx} = \frac{4x - y^3}{6xy^2}$
- D. $\frac{dy}{dx} = \frac{y^3}{3xy^2}$
- E. $\frac{dy}{dx} = \frac{4x - y^3}{3xy^2}$

Question 14

Using an appropriate substitution, $\int \frac{e^{2x}}{1 + e^x} dx$ can be written as

- A. $\int \left(1 - \frac{1}{u}\right) du$
- B. $\int \left(\frac{u}{1 + u^2}\right) du$
- C. $\int \left(\frac{u^2}{1 + u}\right) du$
- D. $\int \left(\frac{2u}{1 + u}\right) du$
- E. $\int \left(1 + \frac{1}{u}\right) du$

Question 15



The direction or slope field for a particular first-order differential equation is shown above. The differential equation could be

- A. $\frac{dy}{dx} = \frac{1}{x+3}$
- B. $\frac{dy}{dx} = \frac{1}{x-3}$
- C. $\frac{dy}{dx} = \log_e(x-3)$
- D. $\frac{dy}{dx} = \log_e(x+3)$
- E. $\frac{dy}{dx} = \sqrt{x-3}$

Question 16

If $\underline{u} = 3\underline{i} - 2\underline{j} + \underline{k}$ and $\underline{v} = \underline{i} - \underline{j} + \underline{k}$, the vector resolute of $\underline{u}$ in the direction of $\underline{v}$ is

- A. $\frac{9}{7}\underline{i} - \frac{6}{7}\underline{j} + \frac{3}{7}\underline{k}$
- B. $2\underline{i} - 2\underline{j} + 2\underline{k}$
- C. $2\underline{i} + 2\underline{j} - 2\underline{k}$
- D. $2\underline{i} - 2\underline{j} - 2\underline{k}$
- E. $\frac{9}{7}\underline{i} + \frac{6}{7}\underline{j} + \frac{3}{7}\underline{k}$

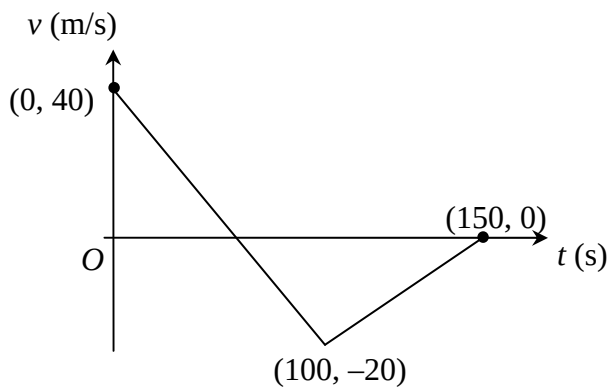
Question 17

A particle moves along a straight line such that its acceleration at time t (seconds) is given by $\ddot{x} = 3t^2 - 4t + 5 \text{ m/s}^2$.

Initially, the particle is at a fixed point, O , ($x = 0$) with a velocity of -2 m/s^2 .

The position of the particle from O at time t is

- A. 6
- B. $\frac{1}{4}t^4 - \frac{2}{3}t^3 + \frac{5}{2}t^2 - 2t$
- C. -6
- D. $3t^4 - \frac{2}{3}t^3 + \frac{5}{2}t^2 - 2t$
- E. $\frac{1}{4}t^4 - t^3 + \frac{5}{2}t^2 - 2t$

Question 18

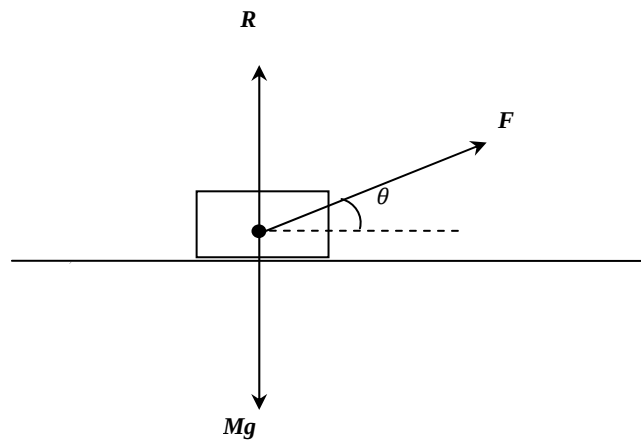
The velocity–time graph of a particle moving in a straight line starting from a fixed position, O , is shown above. If the initial velocity is 40 m/s in an easterly direction, where is the particle located 150 seconds later?

- A. 1000 m west of O
- B. $\frac{6500}{3}$ m east of O
- C. 2000 m east of O
- D. 500 m east of O
- E. 1000 m east of O

Question 19

A body of mass M kg is being pulled along a smooth horizontal table by means of a force, F , acting at an angle θ to the horizontal.

The diagram below indicates the forces acting on the body.



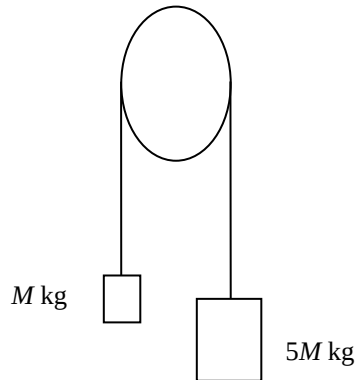
Which statement regarding the magnitude of the forces is true?

- A. $R - Mg = 0$
- B. $R - F \sin \theta - Mg = 0$
- C. $R + F \sin \theta - Mg = 0$
- D. $R + F \cos \theta - Mg = 0$
- E. $R - F \cos \theta - Mg = 0$

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Question 20

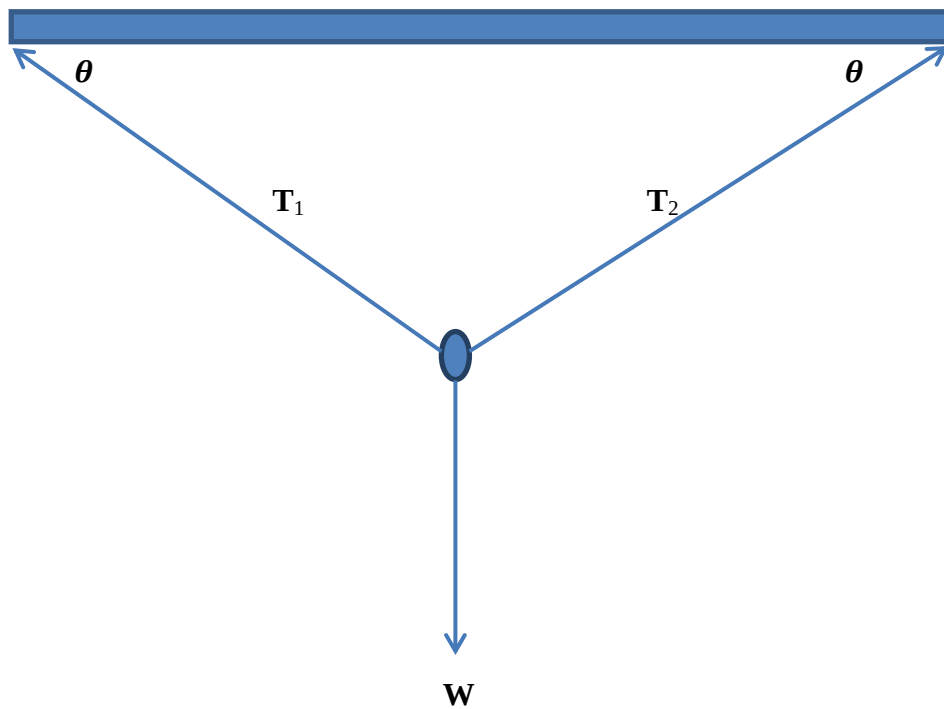
The diagram below shows a smooth pulley with masses of M kg and $5M$ kg attached to each end of an inextensible string.



The magnitude of the acceleration of the $5M$ kg mass is

- A. $\frac{2}{3} \text{ m/s}^2$
- B. $\frac{4g}{3} \text{ m/s}^2$
- C. $\frac{2g}{3} \text{ m/s}^2$
- D. $\frac{4}{3} \text{ m/s}^2$
- E. $3g \text{ m/s}^2$

Question 21



An object in **equilibrium** is suspended from a beam by two ropes, each making an angle of θ with the horizontal. The tension forces acting on the object are T_1 and T_2 and W is the weight force. Which one of the following statements is true?

- A. $T_1 = T_2$
- B. $W = T_1 \cos \theta^\circ + T_2 \cos \theta^\circ$
- C. $T_1 + T_2 = -W$
- D. $T_1 \cos \theta^\circ = T_2 \sin \theta^\circ$
- E. $T_1 + T_2 = 2W$

Question 22

An object of mass 3 kg comes to a complete stop from 20 m/s over a horizontal distance of 60 m. If the acceleration acting on the object is constant, then the **magnitude** of the resultant force acting on the object is

- A. 10 N
- B. $\frac{10}{3}$ N
- C. 19.6 N
- D. -10 N
- E. $\frac{10}{3}$ N

END OF SECTION 1

**END OF SECTION 1
TURN OVER**

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

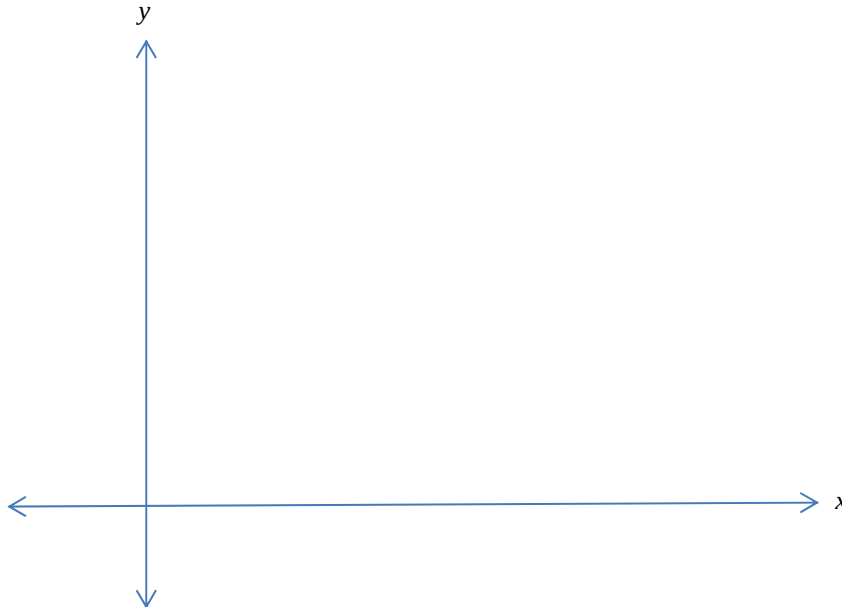
Question 1

Let $f: [a, b] \rightarrow \mathbb{R}$, where $f(x) = (a - x)^2$.

- a. If $f(1) = 0$ and $f(b) = 9$, show that $a = 1$ and $b = 4$.

2 marks

- b. Sketch the graph of the function, clearly labelling the endpoints with their co-ordinates.



1 mark

- c. Find the area enclosed between the graph of the function and the y-axis. (All units are in centimetres.)

2 marks

- d. The function f is rotated about the y-axis to generate a vessel. If the volume of the vessel is $\frac{m\pi}{n} \text{ cm}^3$ find the values of m and n where m and n are both integers.

2 marks

- e. Water now enters the vessel at a constant rate of $2 \text{ cm}^3/\text{s}$. How long does it take for the vessel to be filled to 20% of its capacity?
(Write the answer in minutes, to 2 decimal places.)

2 marks

- f. Write an expression that will determine the height ($h \text{ cm}$) of water in the vessel when it is filled to 20% of its capacity and determine the value of h , to 2 decimal places.

2 marks

Total 11 marks

SECTION 2 – continued
TURN OVER

Question 2

The position of a tugboat relative to an island, O , at any time (t hours) is given by

$$\vec{r}(t) = \left(t + \frac{1}{t}\right)\vec{i} + \left(t - \frac{1}{t}\right)\vec{j}, \quad t > 0,$$

where $\vec{i}$ and $\vec{j}$ are unit vectors east and north of the island, respectively. An oil rig, R , is located 5 km north of the island.

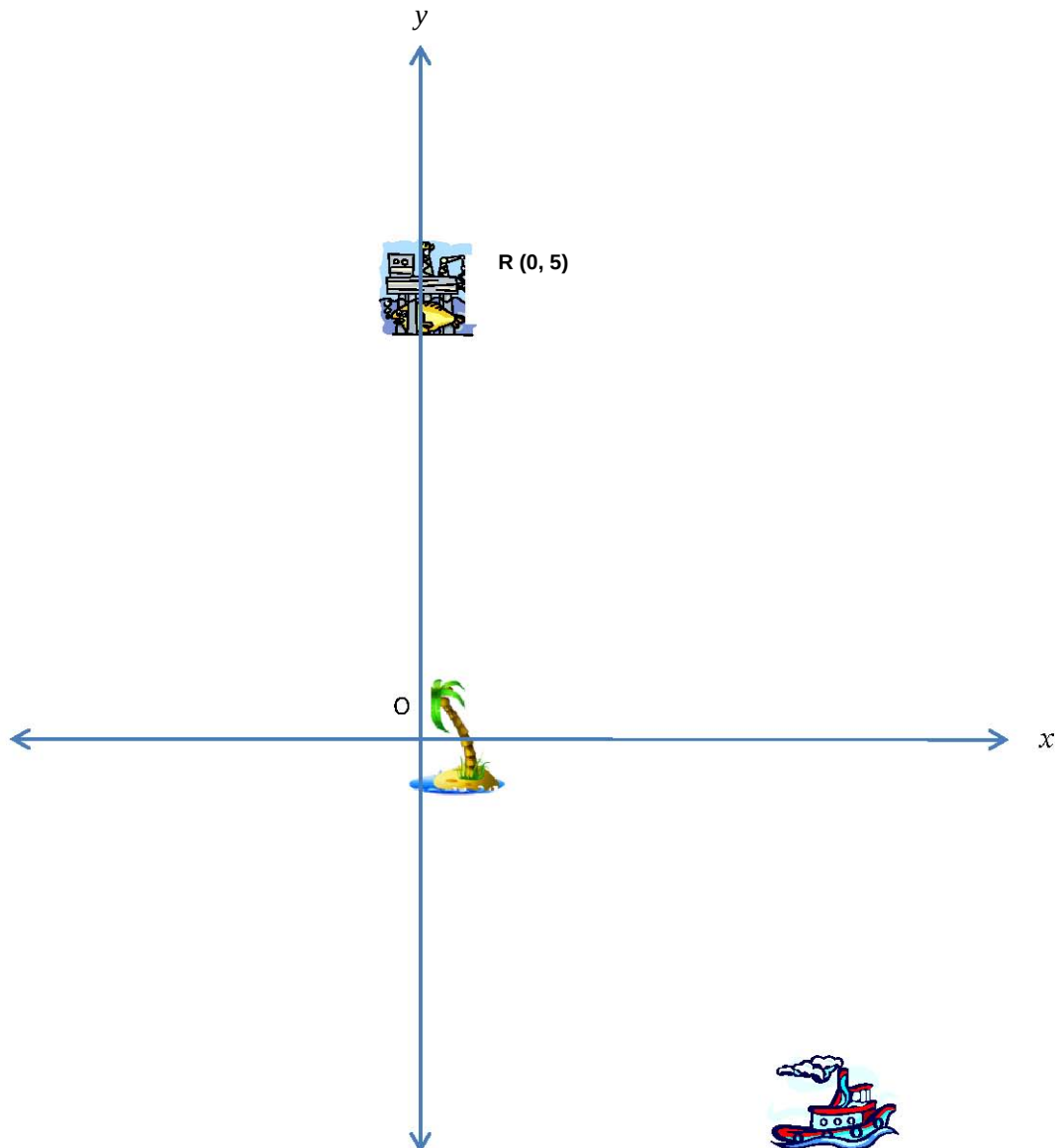
(All distances are in km and the island can be considered as the origin of a Cartesian plane.)

- a.** Show that the path of the tugboat can be described by the Cartesian equation

$$\frac{x^2}{4} - \frac{y^2}{4} = 1 \quad \text{and state the domain and range of the path.}$$

3 marks

- b. Sketch a graph of the path travelled by the tugboat, clearly indicating the direction of motion and label any intercepts.



2 marks

- c. When will the tugboat be due east of the island?

2 marks

- d. Find the velocity vector of the tugboat at any time t .

1 mark

- e. If P is any point on the path of the tugboat, find the vector $\vec{RP}$ in terms of t .

2 marks

- f. Find the time, to the nearest minute, when the tugboat is **closest** to the oil rig.

3 marks

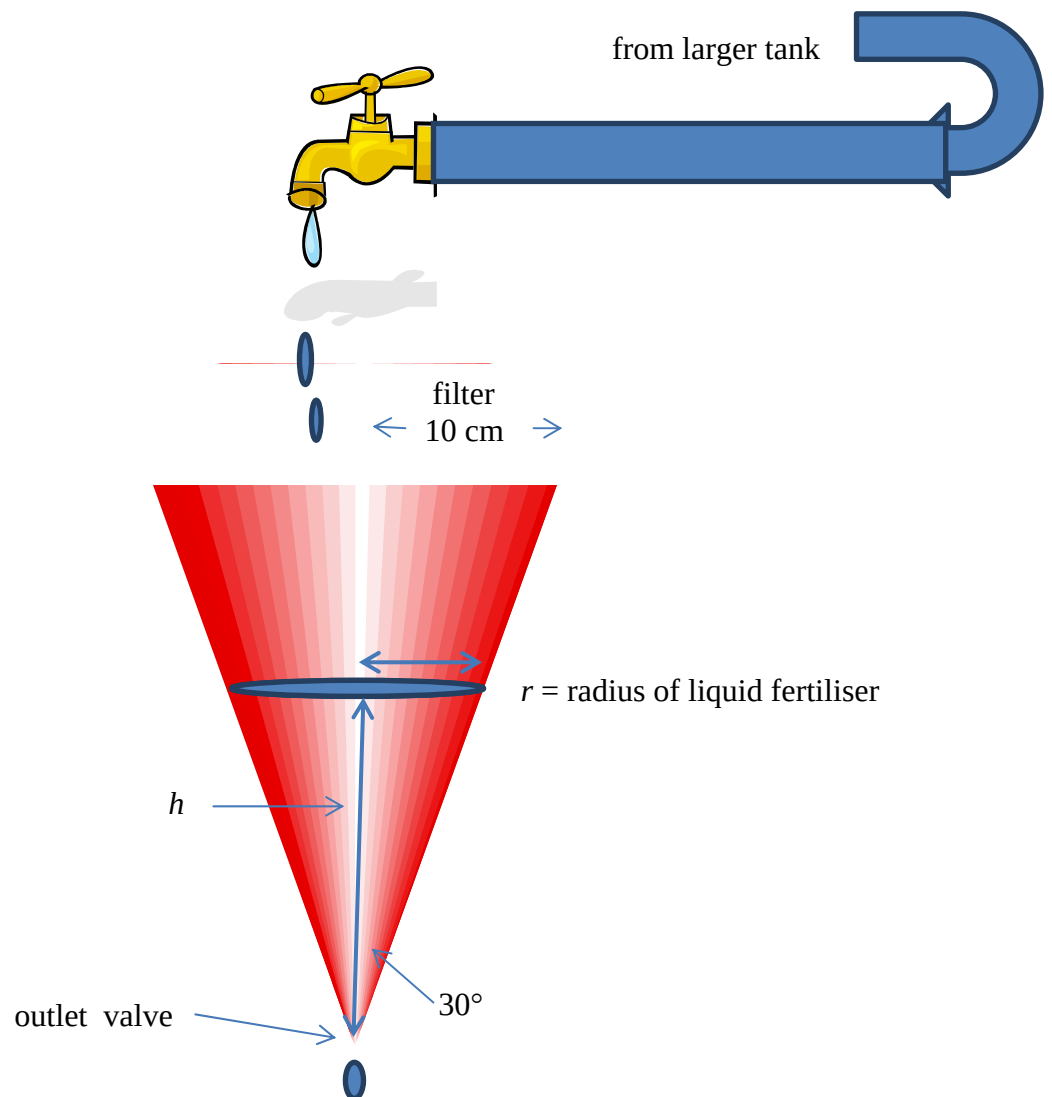
- g.** Find the times, to the nearest minute, when the island, tugboat and oil rig form the vertices of an isosceles triangle with equal sides of 5 km.

4 marks

Total 17 marks

Question 3

A fertiliser drip system contains a filter in the form of an inverted cone with a semi-vertical angle of 30° , as shown in the diagram below. The radius of the filter is 10 cm.



- a.** Liquid fertiliser from a larger tank pours from a tap into the filter at a constant rate of $2 \text{ cm}^3/\text{s}$ and drips from an outlet valve in the bottom at $\frac{\sqrt{h}}{5} \text{ cm}^3/\text{s}$, where $h \text{ cm}$ is the depth of the water in the filter at any time t seconds.

i. Show that $\frac{dV}{dt} = \frac{10 - \sqrt{h}}{5}$.

1 mark

- ii. Find the volume of the liquid fertiliser (V) in terms of its depth (h).

Hence, find $\frac{dh}{dt}$.

[illegible]

4 marks

1 + 4 = 5 marks

- b.** The outlet valve is now closed and the filter is completely filled from the larger tank. Find the **exact** height of liquid fertiliser in the filter when it is completely full.

1 mark

- c. When the filter is completely full, the tap is closed and the outlet valve opened. The liquid fertiliser drips out from the outlet valve at the same rate as before;

$$\text{i.e. } \frac{\sqrt{h}}{5} \text{ cm}^3/\text{s}.$$

- i. Find $\frac{dh}{dt}$ and **use calculus to find t exactly in terms of h .**

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5 marks

- ii.** How long, to the nearest minute, does it take for the filter to empty?

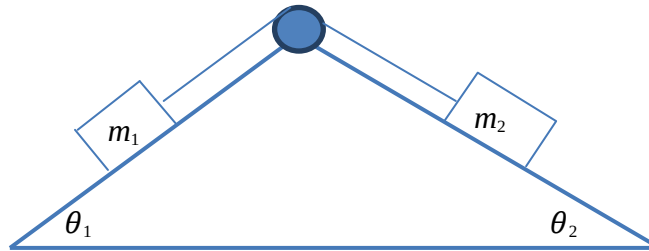
1 mark

5 + 1 = 6 marks

Total 12 marks

Question 4

Two objects of mass m_1 and m_2 are connected by a light string passing over a smooth pulley, which is located at the apex of two inclined planes, as shown in the diagram below. The coefficients of friction of the two surfaces are μ_1 and μ_2 and the angles of inclination are θ_1 and θ_2 , respectively.



Note: The system is on the verge of moving to the left.

- a. Label all the forces acting on the two objects using the symbols $\mathbf{T}$, $\mathbf{N}_1$, $\mathbf{N}_2$, $\mu_1\mathbf{N}_1$, $\mu_2\mathbf{N}_2$, $m_1\mathbf{g}$ and $m_2\mathbf{g}$.

2 marks

- b. The planes are now lubricated and can be considered to be smooth.

- i. If the system is in equilibrium, find $\frac{m_1}{m_2}$ in terms of θ_1 and θ_2 .

3 marks

- ii. The object m_1 is now replaced by an object with a mass that is double that of m_2 and both angles of inclination are the same (θ). Find the acceleration of the system if it is moving to the left.

3 marks

3 + 3 = 5 marks

Total 8 marks

SECTION 2 – continued

TURN OVER

Question 5

a. Consider $u = a - a\sqrt{3}i$, where $a < 0$, and $w = b \operatorname{cis}\left(\frac{\pi}{4}\right)$.

i. Express u in polar form.

2 marks

ii. Express w in Cartesian form.

1 mark

2 + 1 = 3 marks

b. Express each of the following as a complex number.

i. $u + w$ in Cartesian form

1 mark

ii. uw in polar form

1 mark

iii. u^6 in Cartesian form

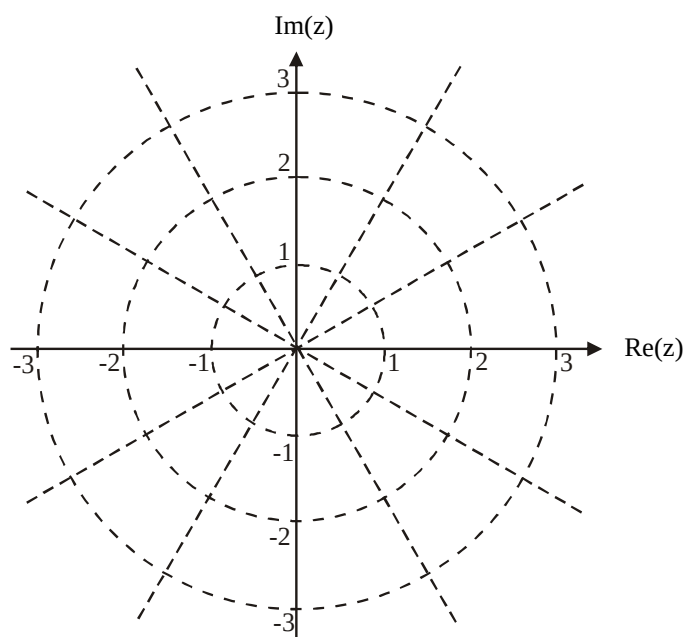
1 mark

1 + 1 + 1 = 3 marks

- c. Find the square roots of w in polar form.

2 marks

- d. If $a = -\frac{1}{2}$ and $b = \sqrt{2}$, plot u^{12} and w^{-2} on the Argand diagram below.



2 marks

Total: $3 + 3 + 2 + 2 = 10$ marks

END OF SECTION 2

END OF QUESTION AND ANSWER BOOK