



INSIGHT
YEAR 12 Trial Exam Paper

2012
FURTHER MATHEMATICS
Written examination 1

Worked Solutions

This book presents:

- correct solutions with full working
- explanatory notes
- tips

This trial examination produced by Insight Publications is NOT an official VCAA paper for the 2012 Further Mathematics written examination 1.

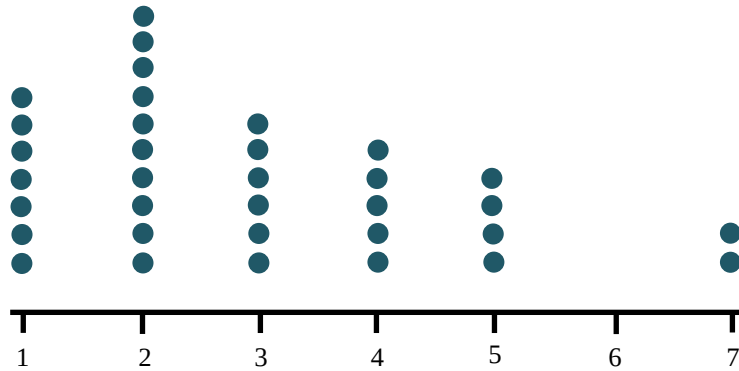
This examination paper is licensed to be printed, photocopied or placed on the school intranet and used only within the confines of the purchasing school for examining their students. No trial examination or part thereof may be issued or passed on to any other party, including other schools, practising or non-practising teachers, tutors, parents, websites or publishing agencies, without the written consent of Insight Publications.

Copyright © Insight Publications 2012

SECTION A**Core: Data analysis**

Use the following information to answer Questions 1, 2 and 3.

The dot plot below shows the distribution of the number of beds in each of 34 households in a retirement village.

**Question 1**

The median of this distribution is

- A. 2
- B. 2.5
- C. 3
- D. 3.5
- E. 4

Answer is B

Worked Solution

There are 34 households in total, so the median will be the $\frac{34 + 1}{2} = 17.5$ th score.

The median is the average of the 17th and 18th values.

Since the 17th value is 2 and the 18th value is 3, the median is

$$\frac{2 + 3}{2} = 2.5$$

Question 2

The percentage of the households that have fewer than three beds is closest to

- A. 68%
- B. 60%
- C. 54%
- D. 50%
- E. 33%

Answer is D

Worked Solution

There are 34 households in total; 17 of them have fewer than three beds.

$$\text{The percentage required} = \frac{17}{34} \times 100\% = 50\%$$

Question 3

Correct to one decimal place, the mean and standard deviation, respectively, of this distribution are

- A. $\bar{x} = 2.9$ $s = 1.7$
 B. $\bar{x} = 2.9$ $s = 1.6$
 C. $\bar{x} = 1.7$ $s = 2.9$
 D. $\bar{x} = 2.5$ $s = 1.2$
 E. $\bar{x} = 2.5$ $s = 1.6$

Answer is A

Worked Solution

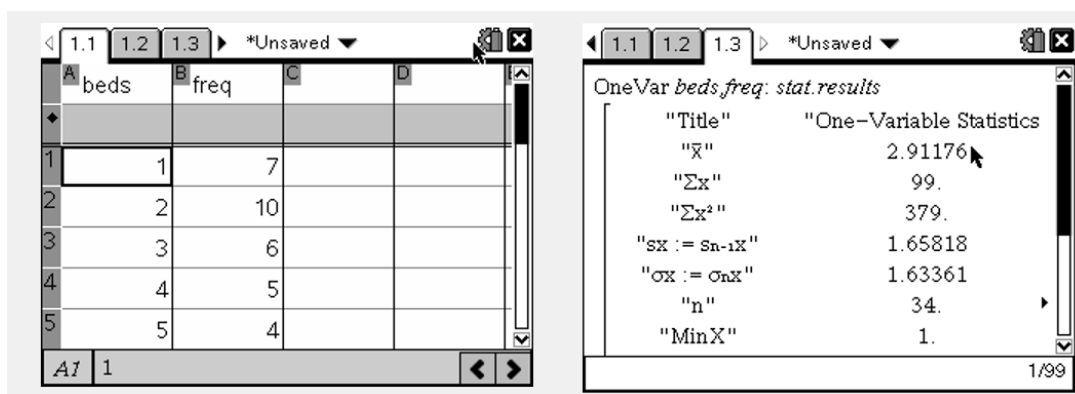
Let's calculate the mean using the following formula.

$$\bar{x} = \frac{\sum xf}{\sum f}$$

$$\bar{x} = \frac{1 \times 7 + 2 \times 10 + 3 \times 6 + 4 \times 5 + 5 \times 4 + 7 \times 2}{7 + 10 + 6 + 5 + 4 + 2}$$

$$\bar{x} = \frac{99}{34} = 2.91176 \approx 2.9$$

Alternatively, we can find the solution by calculator.



Question 4

The variables

salary (high, medium, low)
time spent watching television (hours)

- A. are both numerical.
- B. are both categorical.
- C. are numerical and categorical, respectively.
- D. are categorical and numerical, respectively.**
- E. are neither categorical nor numerical.

Answer is D

Worked Solution

The variable *salary* is a categorical variable since it is recorded as high, medium or low (i.e. three categories) instead of as an amount of dollars. The variable *time spent watching television* is a numerical variable because it is recorded in hours.

Use the following information to answer Questions 5 and 6.

The heights of tulips in a botanic garden have a normal distribution with a mean of 42 cm and a standard deviation of 6.6 cm.

Question 5

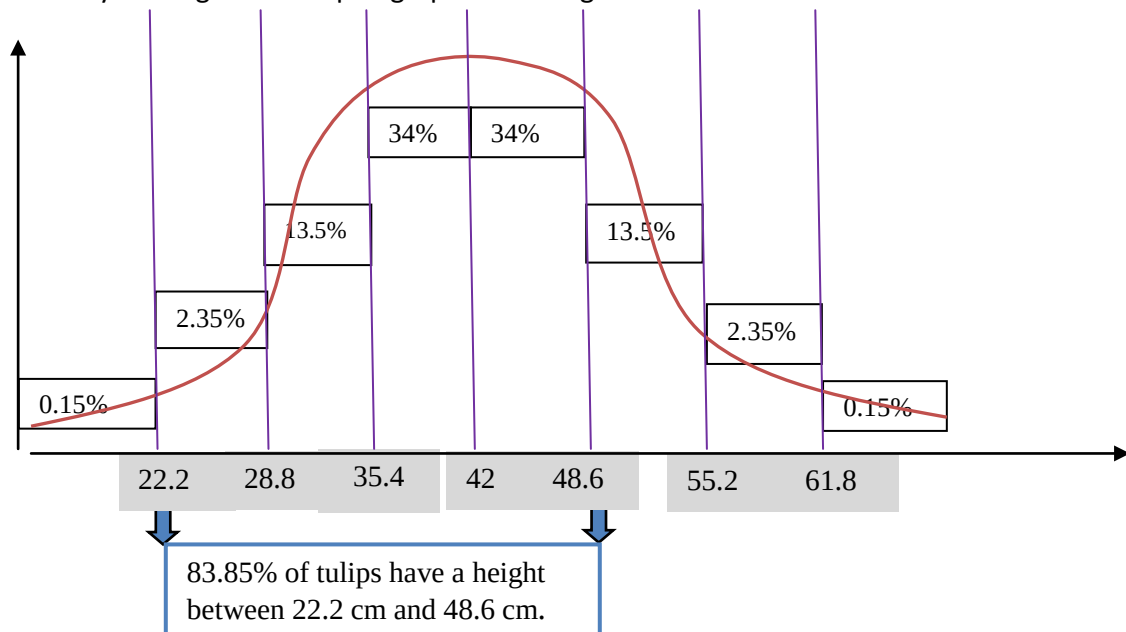
The percentage of tulips that have a height between 22.2 cm and 48.6 cm is

- A. 99.7%
- B. 95%
- C. 81.5%
- D. 68%
- E. 83.85%**

Answer is E

Worked Solution

We'll start by making a bell-shaped graph with the given mean and standard deviation values.



$$2.35 + 13.5 + 34 + 34 = 83.85\%$$

Question 6

Anne bought a tulip from the garden shop. The tulip that Anne bought has a standardised height of $z = 2.5$. This corresponds to an actual tulip height of

- A. 48.6 cm
- B. 55.2 cm
- C. **58.5 cm**
- D. 60 cm
- E. 61.8 cm

Answer is C

Worked Solution

$$z = \frac{x - \bar{x}}{S_x}$$

$$2.5 = \frac{x - 42}{6.6}$$

$$x = 58.5 \text{ cm}$$

Question 7

In an investigation to find the relationship between the food energy content (in calories) and the carbohydrate content (in grams) of 20 different brands of standard-sized packets of biscuits (each packet of biscuits being 100 grams), the following summary statistics were obtained.

the total carbohydrate content = 1504 grams

the total energy content = 11 600 calories

standard deviation of carbohydrate content = 3.5 grams

standard deviation of energy content = 32 calories

Pearson's correlation coefficient = 0.86

In this investigation, energy content is the dependent variable. The y-intercept of the corresponding least squares regression line is

- A. 0.86
- B. -7.86
- C. 7.86
- D. 11.29
- E. **-11.29**

Answer is E

Worked Solution

Let's calculate mean values for the total carbohydrate content ($\bar{x}$) and the total energy content ($\bar{y}$).

$$\bar{x} = \frac{\text{the total carbohydrate content}}{\text{number of biscuit brands}} = \frac{1504}{20} = 75.2$$

$$\bar{y} = \frac{\text{the total energy content}}{\text{number of biscuit brands}} = \frac{11\,600}{20} = 580$$

Now let's calculate the gradient and the y-intercept of the equation $y = c + mx$.

$$\text{the gradient} = m = r \times \frac{S_y}{S_x} = 0.86 \times \frac{32}{3.5} = 7.862\,857$$

$$\text{the y-intercept} = c = \bar{y} - m \times \bar{x} = 580 - 7.862\,857 \times 75.2 = -11.29$$

Use the following information to answer Questions 8, 9 and 10.

The following data give the heights (in centimetres) and weights (in kilograms) of nine people.

Height (in cm)	168	155	147	175	172	158	182	190	161
Weight (in kg)	58	50	45	70	65	51	83	102	54

After analysing the data, the least squares regression equation to predict the weights of these people from their heights is found to be

$$\text{weight} = -151.0346 + 1.2847 \times \text{height}$$

Question 8

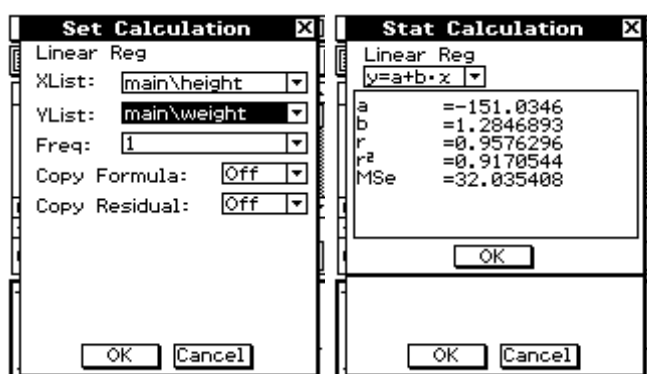
From the height and weight measurements given in the table above, it can be concluded that the percentage of the variation in *weights* of the people that can be explained by the variation in their *heights* is closest to

- A. 95.8%
- B. 99.6%
- C. 85.3%
- D. **91.7%**
- E. 75.4%

Answer is D

Worked Solution

The percentage of the variation in *weights* of the people that can be explained by the variation in their *heights* is the coefficient of determination, which is the r^2 value. Let's calculate this value together, using a calculator.



So $r^2 = 91.7\%$.

Question 9

The residual for a height of 182 cm is closest to

- A. -0.2
- B. 82.8
- C. **0.2**
- D. 83
- E. -82.8

Answer is C

Worked Solution

$$y_{\text{predicted}} = -151.0346 + 1.2847 \times 182 = 82.7808$$

$$\text{residual} = y_{\text{actual}} - y_{\text{predicted}} = 83 - 82.7808 = 0.2$$

Question 10

After analysing the residual plot, the relationship between *weights* and *heights* of these people is found to be non-linear. In an attempt to linearise the data, a reciprocal transformation is applied to the variable *weight*.

The equation of the least squares regression line that allows the reciprocal of the *weights* of these people to be predicted from their *heights* is closest to

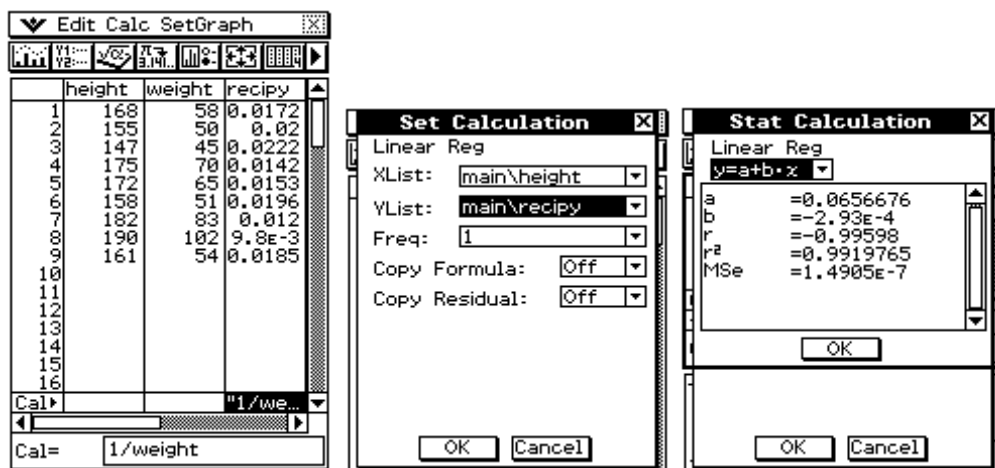
- A. $\text{weight} = 0.07 - 2.93 \times \text{height}$
- B. $\frac{1}{\text{weight}} = 0.0657 - 0.0003 \times \text{height}$
- C. $\text{weight} = 0.0657 - 0.0003 \times \text{height}$
- D. $\frac{1}{\text{weight}} = 0.07 - 2.93 \times \text{height}$
- E. $\text{weight} = 0.07 - \frac{2.93}{\text{height}}$

Answer is B

Worked Solution

This is a calculator exercise. It must be noted that *height* is the independent variable and $\frac{1}{\text{weight}}$ is the dependent variable.

Use a calculator.



$$\frac{1}{\text{weight}} = 0.0657 - 0.0003 \times \text{height}$$

Use the following information to answer Questions 11 and 12.

The seasonal index for hay fever tablet sales by a chemist shop during spring is 1.6.

Question 11

The hay fever tablet sales that the chemist shop makes during spring is typically

- A. 40% above the quarterly average.
- B. 60% below the quarterly average.
- C. 37.5% below the quarterly average.
- D. 60% above the quarterly average.**
- E. 37.5% above the quarterly average.

Answer is D

Worked Solution

The seasonal index for hay fever tablet sales during spring is 1.6, which corresponds to 160%. Since the quarterly average is 100%, the hay fever tablet sales that the chemist makes during spring is $160\% - 100\% = 60\%$ above the quarterly average.

Question 12

To correct for seasonality, the hay fever tablet sales figures for spring should be

- A. increased by 60%.
- B. increased by 37.5%.
- C. reduced by 60%.
- D. reduced by 40%.
- E. reduced by 37.5%.**

Answer is E

Worked Solution

Let us assume that 100 hay fever tablets are sold during spring.

$$\text{deseasonalised value} = \frac{\text{actual value}}{\text{seasonal index}}$$

$$\text{deseasonalised value} = \frac{100}{1.6} = 62.5$$

$100\% - 62.5\% = 37.5\%$ reduction to be applied in order to correct for seasonality

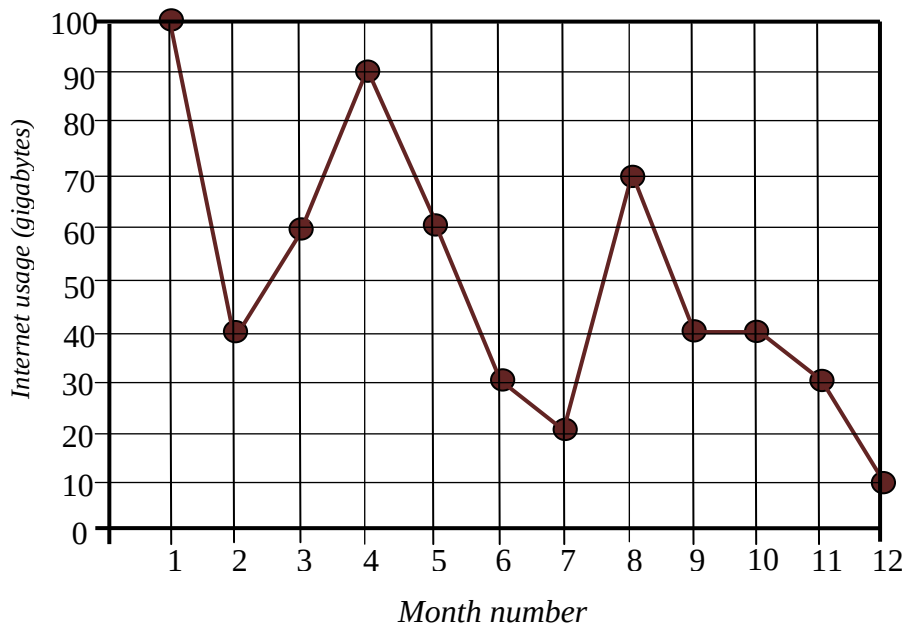


Tip

- A common mistake that students make is to increase the sales by 60%. A seasonal index of 1.6 means that the hay fever tablet sales during spring are 60% above the quarterly average. Since the hay fever tablet sales during spring are higher than the quarterly average, we need to reduce this value in order to correct for seasonality.

Question 13

The data below show the internet usage (in gigabytes of information downloaded) by a computer at an internet café in Melbourne from January to December.



The **six** median smoothed internet usage value centred at the seventh month is

- A. 45
- B. 40
- C. 30
- D. 20
- E. 47

Answer is A

Worked Solution

Let's smooth the data.

Month number	1	2	3	4	5	6	7	8	9	10	11	12
Internet usage (GB)	100	40	60	90	60	30	20	70	40	40	30	10

The median of
20, 30, 40, 60, 70, 90

50

50 40

The median of
20, 30, 40, 40, 60, 70

40

$$\frac{50 + 40}{2} = 45$$

Six median smoothed internet usage centred at the seventh month is 45 gigabytes.

END OF SECTION A
TURN OVER

SECTION B**Module 1: Number patterns****Question 1**

For the sequence $-3, 4, 11, 18, \dots$ the sum of the first 15 terms is

- A. 450
- B. **690**
- C. 300
- D. 960
- E. 470

Answer is B

Worked Solution

$$t_2 - t_1 = 4 - (-3) = 7$$

$$t_3 - t_2 = 11 - 4 = 7$$

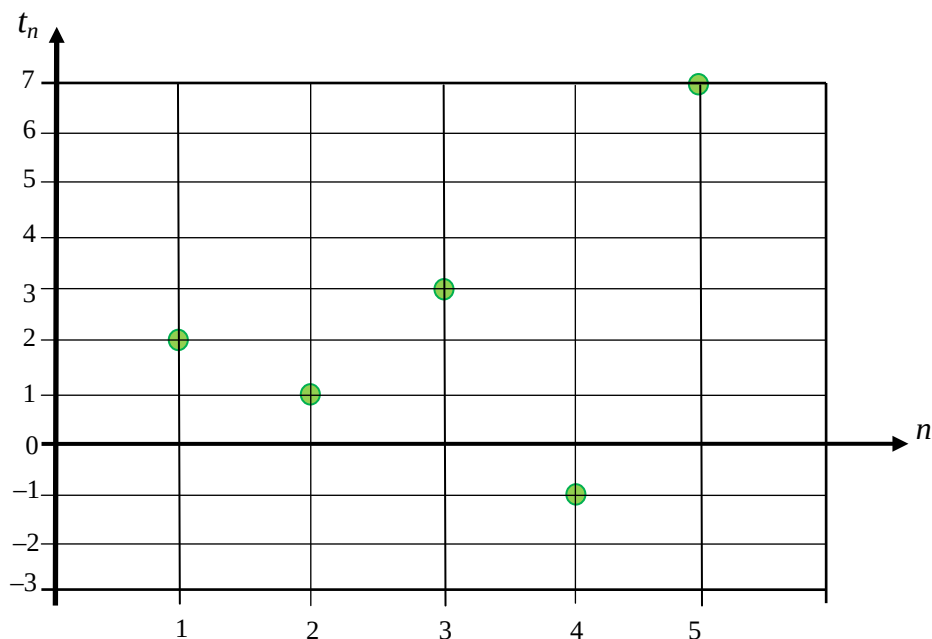
Since $t_2 - t_1 = t_3 - t_2 = 7$, the sequence is arithmetic with a common difference of 7.

Let's use the arithmetic sum formula to calculate the sum of the first fifteen terms of the sequence.

$$S_n = \frac{n}{2} [2a + (n-1)d] \quad \text{where } a = -3, d = 7 \text{ and } n = 15$$

$$S_{15} = \frac{15}{2} [2 \times -3 + (15-1)7]$$

$$S_{15} = 690$$

Question 2

The first five terms of a sequence are plotted on the graph above. Which of the following difference equations matches the sequence in the graph?

- A.** $t_{n+1} = t_n - 1$ where $t_1 = 2$
B. $t_{n+1} = 2t_n - 3$ where $t_1 = 2$
C. $t_{n+1} = -t_n + 3$ where $t_1 = 2$
D. $t_{n+1} = -2t_n + 5$ where $t_1 = 2$
E. $t_{n+1} = 3t_n - 5$ where $t_1 = 2$

Answer is D

Worked Solution

The first five terms of the sequence are 2, 1, 3, -1, 7.

Generate the sequences in the options from the difference equations.

- Option A: $t_2 = t_1 - 1 = 2 - 1 = 1$
 $t_3 = t_2 - 1 = 1 - 1 = 0 \neq 3$ So option A cannot be the correct answer.
- Option B: $t_2 = 2t_1 - 3 = 2 \times 2 - 3 = 1$
 $t_3 = 2t_2 - 3 = 2 \times 1 - 3 = -1 \neq 3$ So option B cannot be the correct answer.
- Option C: $t_2 = -t_1 + 3 = -2 + 3 = 1$
 $t_3 = -t_2 + 3 = -1 + 3 = 2 \neq 3$ So option C cannot be the correct answer.
- Option D: $t_2 = -2t_1 + 5 = -2 \times 2 + 5 = 1$
 $t_3 = -2t_2 + 5 = -2 \times 1 + 5 = 3$ So option D is the correct answer.
- Option E: $t_2 = 3t_1 - 5 = 3 \times 2 - 5 = 1$
 $t_3 = 3t_2 - 5 = 3 \times 1 - 5 = -2 \neq 3$ So option E cannot be the correct answer.

Question 3

The sixth, tenth and fourteenth terms of a sequence are 3, 27 and 243, respectively. This sequence

- A. must be arithmetic with the seventh term being 9.
- B. must be geometric with the seventh term being $3\sqrt{3}$.
- C. **can be geometric with the seventh term being $3\sqrt{3}$, but is not necessarily so.**
- D. can be arithmetic with the seventh term being 9, but is not necessarily so.
- E. cannot be either arithmetic or geometric.

Answer is C

Worked Solution

For the sequence to be arithmetic, $t_{10} - t_6$ must be equal to $t_{14} - t_{10}$.

$$t_{10} - t_6 = 27 - 3 = 24$$

$$t_{14} - t_{10} = 243 - 27 = 216$$

Since $t_{10} - t_6 \neq t_{14} - t_{10}$, the sequence is not arithmetic.

For the sequence to be geometric there must be a common ratio r , between the terms.

Hence, $\frac{t_{10}}{t_6}$ must be equal to $\frac{t_{14}}{t_{10}}$.

$$\frac{t_{10}}{t_6} = \frac{27}{3} = 9 \quad \text{and} \quad \frac{t_{14}}{t_{10}} = \frac{243}{27} = 9$$

Since $\frac{t_{10}}{t_6} = \frac{t_{14}}{t_{10}}$, the sequence can be geometric.

$$\frac{t_{10}}{t_6} = 9$$

$$\frac{ar^{10-1}}{ar^{6-1}} = 9$$

$$r^4 = 9$$

$$r = \pm\sqrt{3}$$

$$t_7 = t_6 \times r = 3 \times \pm\sqrt{3} = \pm 3\sqrt{3}$$

The seventh term could be $3\sqrt{3}$, but it could also be $-3\sqrt{3}$.

Question 4

The number of mangoes in the bottom layer of a huge fruit display is 100. Each successive layer contains three fewer mangoes than the layer below.

This pattern of stacking mangoes in layers continues.

The maximum number of mangoes that can be stacked in this way is

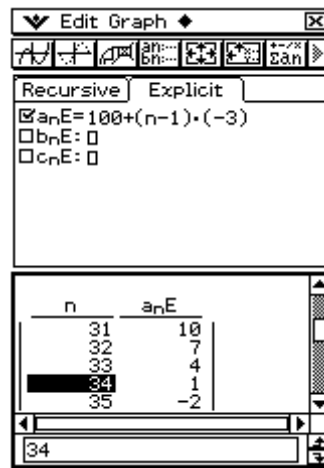
- A. 34
- B. 3400
- C. 1695
- D. 2520
- E. 1717

Answer is E

Worked Solution

The sequence that is formed by the number of mangoes in each successive layer of the fruit display is arithmetic. The first term of this sequence is 100 and the common difference is -3 .

Use a calculator.



There are 34 layers of mangoes in this display. Let's use the arithmetic sum formula to calculate S_{34} .

$$s_n = \frac{n}{2} [2a + (n-1)d]$$

$$s_{34} = \frac{34}{2} [2 \times 100 + (34-1) \times -3]$$

$$s_{34} = 1717$$

Question 5

For which one of the following geometric sequences is an infinite sum able to be determined?

- A. $\frac{1}{81}, -\frac{1}{27}, \frac{1}{9}, -\frac{1}{3}, \dots$
- B. $1, 3, 9, 27, \dots$
- C. $-1, 3, -9, 27, \dots$
- D. $27, -9, 3, -1, \dots$
- E. $\frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \dots$

Answer is D

Worked Solution

The infinite sum of a geometric sequence can only be determined when the common ratio is between -1 and 1 . Let's determine the common difference in all of the five options.

Option A: $r = \frac{t_2}{t_1} = \frac{\frac{-1}{27}}{\frac{1}{81}} = -3$ Infinite sum cannot be determined.

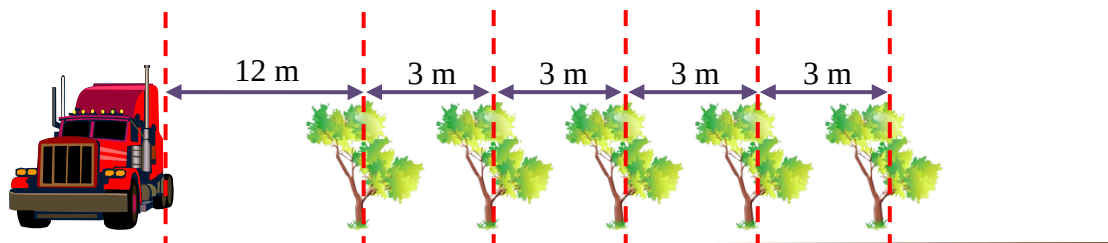
Option B: $r = \frac{t_2}{t_1} = \frac{3}{1} = 3$ Infinite sum cannot be determined.

Option C: $r = \frac{t_2}{t_1} = \frac{3}{-1} = -3$ Infinite sum cannot be determined.

Option D: $r = \frac{t_2}{t_1} = \frac{-9}{27} = -\frac{1}{3}$ Since r is between -1 and 1 , infinite sum can be determined.

Option E: $r = \frac{t_2}{t_1} = \frac{\frac{1}{27}}{\frac{1}{81}} = 3$ Infinite sum cannot be determined.

Use the following information to answer Questions 6 and 7.



Adam, the farmer, has some trees to be planted on a line, 3 metres apart. The truck containing trees is 12 metres from the point at which the first tree is to be planted.

The trees are very heavy and Adam can carry only one at a time. As a result he has to collect the first tree from the truck, carry it to its position, plant it, and return to the truck. Then he has to get the second tree, carry it to its position, plant it, return to the truck, get the third tree and so on.

Question 6

If D_n is the distance from the truck to the n th tree and back to the start, a difference equation that can be used to model the distance is

- | | | | |
|----|----------------------|-------|------------|
| A. | $D_{n+1} = D_n + 3$ | where | $D_1 = 12$ |
| B. | $D_{n+1} = D_n + 15$ | where | $D_1 = 12$ |
| C. | $D_{n+1} = D_n + 6$ | where | $D_1 = 24$ |
| D. | $D_{n+1} = 3D_n$ | where | $D_1 = 12$ |
| E. | $D_{n+1} = 6D_n + 6$ | where | $D_1 = 24$ |

Answer is C

Worked Solution

This is an arithmetic sequence with a common difference of 6 and a first term of 24. The difference equation representing this sequence is $D_{n+1} = D_n + 6$, where $D_1 = 24$.



Tip

- An answer of A would have been obtained if the doubling of 12 and 3 were forgotten. Whenever Adam carries a tree to its position he must return to the truck to get the next tree, so we need to double the initial distances.

Question 7

In carrying out his work, Adam walks at least 1.2 kilometres a day while planting trees. Find the minimum number of trees he would need to plant in order for the total distance he walks while planting these trees to be at least 1.2 kilometres.

- A. 27
- B. 16
- C. 10
- D. 14
- E. 17

Answer is E

Worked Solution

First, we need to write the arithmetic sum formula.

$$s_n = \frac{n}{2} [2a + (n-1)d]$$

$$s_n = \frac{n}{2} [2 \times 24 + (n-1)6]$$

We can equate this formula to 1.2 kilometres (which equals 1200 metres) and solve for n .

$$\frac{n}{2} [2 \times 24 + (n-1)6] = 1200$$

$$n = 16.8 \approx 17 \text{ trees}$$

Alternatively we can produce the table of values on a calculator to determine the number of trees.

n	a_nE
14	882
15	990
16	1104
17	1224
18	1350

Question 8

Serra has taken out a \$400 000 loan to purchase an investment property. The amount of money she owes to the bank at the start of the n th year is found to follow the difference equation

$$M_{n+1} = 1.1M_n - 75\,500$$

This is consistent with

- A. 1% interest being charged each year and then Serra depositing \$75 500 in the bank.
- B. Serra withdrawing \$75 500 from the bank each year and then 1% interest being charged.
- C. 9% interest being charged each year and then Serra depositing \$75 500 in the bank.
- D. 10% interest being charged each year and then Serra withdrawing \$75 500 from the bank.
- E. **10% interest being charged each year and then Serra depositing \$75 500 in the bank.**

Answer is E

Worked Solution

In order to get the next term of the difference equation, 1.1 is multiplied by the previous term.

$$1.1 \times 100\% = 110\%$$

$$110\% - 100\% = 10\% \text{ increase in Serra's debt}$$

This shows that 10% interest is being charged yearly.

Then, according to the formula, the debt Serra owes to the bank decreases by \$75 500.

This shows that Serra deposits \$75 500 in the bank each year.

Question 9

Tony has a rabbit farm where he breeds and sells rabbits. He realises that the number of rabbits after n months can be modelled by the difference equation

$$R_n = 3R_{n-1} - R_{n-2} \quad \text{where } R_0 = 4 \text{ and } R_1 = 3$$

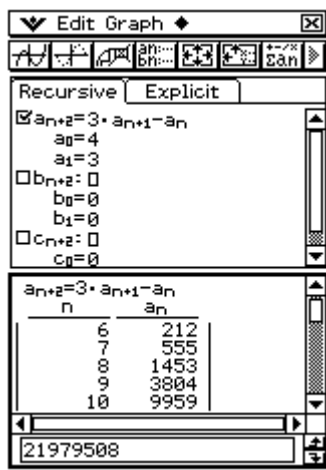
The number of rabbits that Tony has after nine months is

- A. 1453
- B. **3804**
- C. 9959
- D. 981
- E. 198

Answer is B

Worked Solution

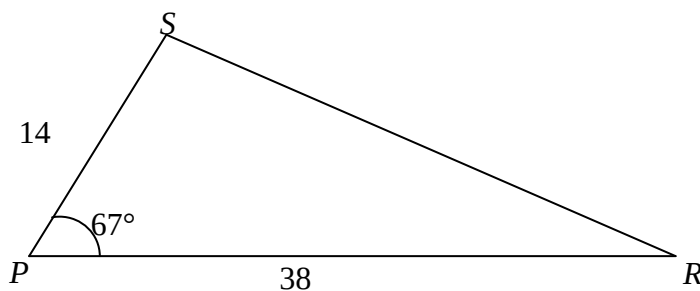
Use a calculator.



The number of rabbits that Tony had after nine months is 3804.

Module 2: Geometry and trigonometry

Use the following information to answer Questions 1, 2 and 3.



In triangle PRS , $\angle SPR = 67^\circ$.

The length of side $PS = 14$ cm and $PR = 38$ cm.

Question 1

The area of triangle PRS in centimetres squared is closest to

- A. 245
- B. 233
- C. 104
- D. 142
- E. 358

Answer is A

Worked Solution

The side opposite angle R is r .

The side opposite angle S is s .

The side opposite angle P is p .

$$\text{Area} = \frac{1}{2} \times r \times s \times \sin P$$

$$\text{Area} = \frac{1}{2} \times 14 \times 38 \times \sin 67^\circ$$

$$\text{Area} = 244.85 \text{ cm}^2 \cong 245 \text{ cm}^2$$

Question 2

The length of side SR is closest to

- A. 24.35 cm
- B. 28.58 cm
- C. **34.99 cm**
- D. 39.85 cm
- E. 42.22 cm

Answer is C

Worked Solution

We will evaluate the length of side SR using the cosine rule.

$$SR = p$$

$$\text{The cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{In this case: } p^2 = r^2 + s^2 - 2rs \cos P$$

$$p^2 = 14^2 + 38^2 - 2 \times 14 \times 38 \times \cos 67^\circ$$

$$p = 34.9895 \cong 34.99 \text{ cm}$$

Question 3

The magnitude of $\angle PSR$ is closest to

- A. 73°
- B. 80°
- C. 85°
- D. **89°**
- E. 94°

Answer is D

Worked Solution

We will evaluate the magnitude of $\angle PSR$ using the sine rule.

$PSR = \theta$

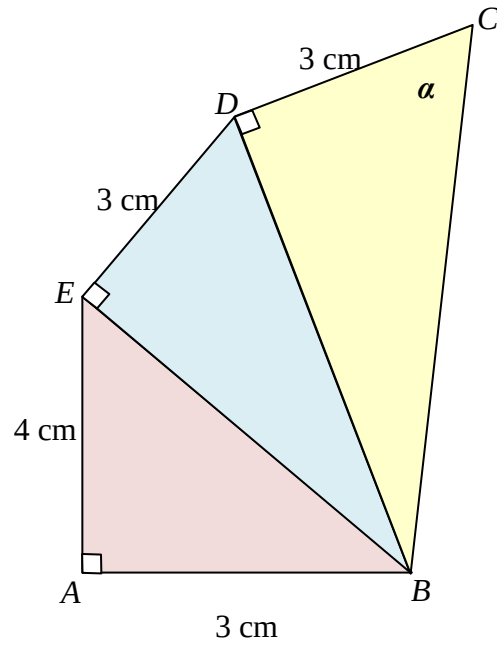
The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

In this case: $\frac{s}{\sin S} = \frac{p}{\sin P} = \frac{r}{\sin R}$

If $\frac{s}{\sin S} = \frac{p}{\sin P}$: $\frac{38}{\sin \theta} = \frac{34.99}{\sin 67^\circ}$

$\theta = 88.6^\circ \cong 89^\circ$

Question 4



For the triangles in the diagram above, the magnitude of angle α is closest to

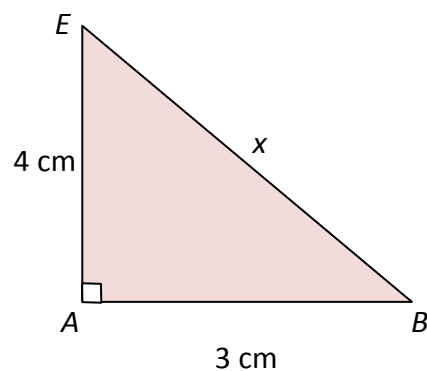
- A. 27°
- B. 55°
- C. 94°
- D. **63°**
- E. 58°

Answer is D

Worked Solution

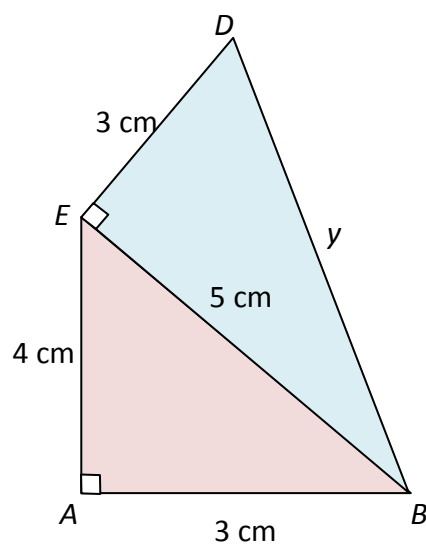
Step 1: Let's evaluate the length of side EB using Pythagoras' theorem.

$$\begin{aligned} EB &= x \\ x^2 &= 4^2 + 3^2 \\ x &= 5 \text{ cm} \end{aligned}$$



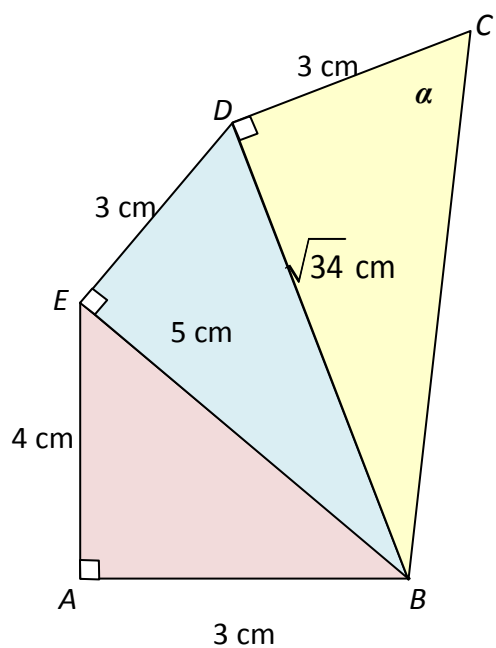
Step 2: Let's evaluate the length of side DB using Pythagoras' theorem.

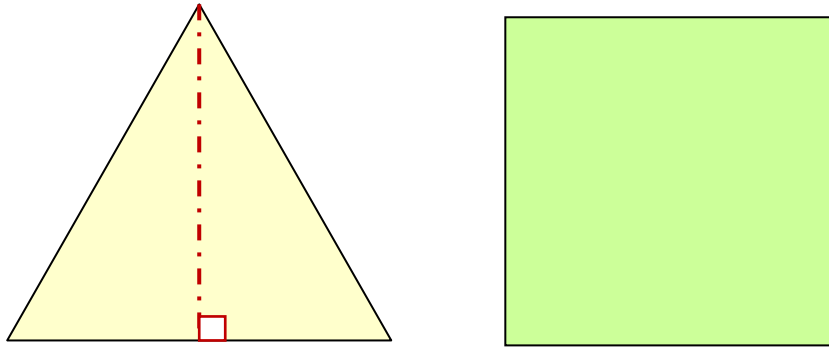
$$\begin{aligned} DB &= y \\ y^2 &= 3^2 + 5^2 \\ y^2 &= 34 \\ y &= \sqrt{34} \text{ cm} \end{aligned}$$



Step 3: Let's use the tangent ratio in triangle BCD in order to determine the magnitude of angle α .

$$\begin{aligned} \tan \alpha &= \frac{\sqrt{34}}{3} \\ \alpha &= \tan^{-1}\left(\frac{\sqrt{34}}{3}\right) \\ \alpha &= 62.77^\circ \cong 63^\circ \end{aligned}$$



Question 5

The square shown above has an area of 64 cm^2 .

The height of the equilateral triangle shown above is equal to the side length of the square.

The area of the triangle is closest to

- A. 8 cm^2
- B. 32 cm^2
- C. **36.95 cm^2**
- D. 38.57 cm^2
- E. 18.48 cm^2

Answer is C

Worked Solution

Since the area of the square is 64 cm^2 , the side length of this square is equal to $\sqrt{64} \text{ cm}$ which is 8 cm .

Now let's evaluate the side length (base) of the equilateral triangle using the tangent ratio.

Let the length of the base equal x .

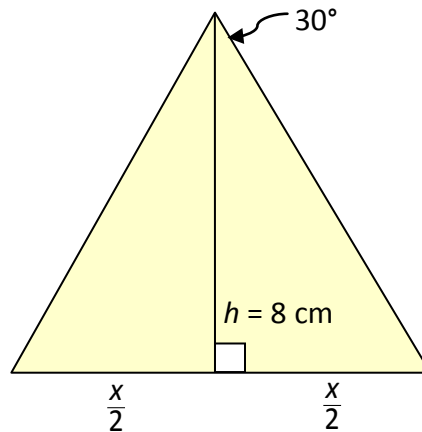
$$\tan 30^\circ = \frac{\frac{x}{2}}{8}$$

$$x = 9.237\ 604\ 307 \text{ cm}$$

$$\text{Area of the triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area of the triangle} = \frac{1}{2} \times 9.237\ 604\ 307 \times 8$$

$$\text{Area of the triangle} = 36.95 \text{ cm}^2$$



Question 6

Members of a group of students on school camp go bushwalking from their campsite (C) to a river (R) along a straight fire track for a distance of 4 km on a bearing of 65° .

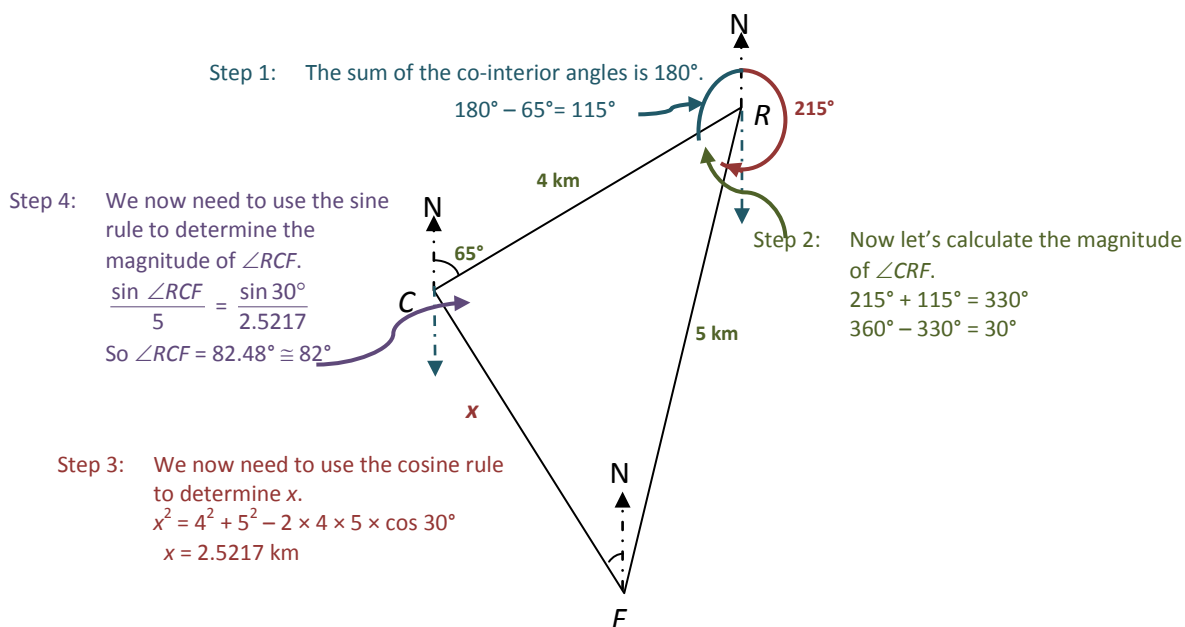
They then walk along another straight fire track for a distance of 5 km on a bearing of 215° . They stop for fishing at this spot (F). The bearing of the fishing spot from the original campsite at C is closest to

- A. 138°T
- B. 147°T
- C. 115°T
- D. 082°T
- E. 115°T

Answer is B

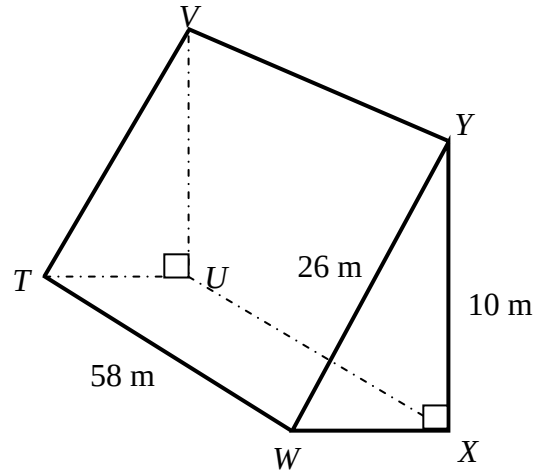
Worked Solution

Let's start solving the problem by drawing a diagram.



So, the bearing of point F from point C is
 $65^\circ + 82^\circ = 147^\circ\text{T}$

Use the following information to answer Questions 7 and 8.



The storage container above is a prism with a cross-section in the shape of a right-angled triangle.

As can be seen from the diagram above, the edges are the following lengths.

$$TW = 58 \text{ m}$$

$$XY = 10 \text{ m}$$

$$WY = 26 \text{ m}$$

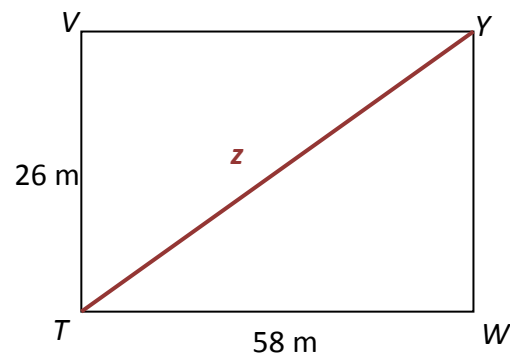
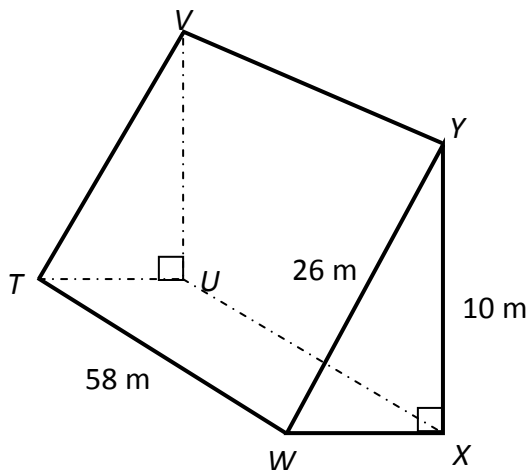
Question 7

A cable is to connect directly from point T to point Y . The length of the cable is closest to

- A. 58.86 m
- B. 59.23 m
- C. 42 m
- D. 64.34 m
- E. 63.56 m

Answer is E

Worked Solution



Let's use Pythagoras' theorem to evaluate the length of the cable.

$$z^2 = 58^2 + 26^2$$

$$z = 63.56 \text{ m}$$



Tip

- An answer of A would have been obtained if 10 was used instead of 26 as one of the edges.

Question 8

The volume of the storage container is

- A. 10 450 m³
- B. 8590 m³
- C. 7540 m³
- D. **6960 m³**
- E. 5436 m³

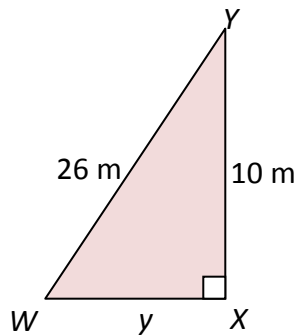
Answer is D

Worked Solution

First we need to evaluate the length of edge WX using Pythagoras' theorem.

$$y^2 = 26^2 - 10^2$$

$$y = 24 \text{ m}$$



Volume of the prism = Area of cross-section $\times$ height (or length)

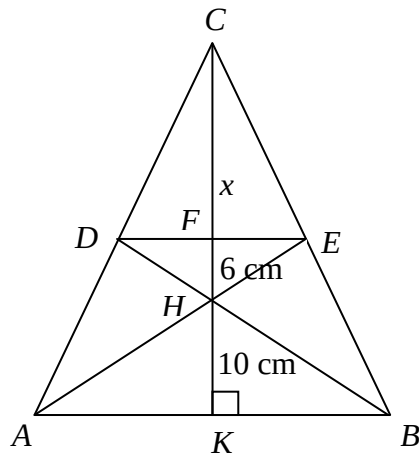
$$V = A \times h$$

$$V = \frac{1}{2} \times 24 \times 10 \times 58$$

$$V = 6960 \text{ m}^3$$

**Tip**

- An answer of C would have been obtained if 26 was used instead of 24 in the volume formula.

Question 9

In triangle ABC shown above, $DE \parallel AB$ and $CK \perp AB$.

$FH = 6 \text{ cm}$ and $HK = 10 \text{ cm}$.

The length of CF , in cm, is closest to

- A. 8
- B. 9.6
- C. 12
- D. 20.2
- E. 24

Answer is E

Worked Solution

Triangle DFH is similar to triangle BKH .

$$\frac{DH}{HB} = \frac{FH}{KH}$$

$$\frac{DH}{HB} = \frac{6}{10}$$

Triangle DEH is similar to triangle BAH .

$$\frac{DE}{AB} = \frac{DH}{HB}$$

$$\frac{DE}{AB} = \frac{6}{10}$$

Triangle CDE is similar to triangle CAB .

$$\frac{CD}{CA} = \frac{DE}{AB}$$

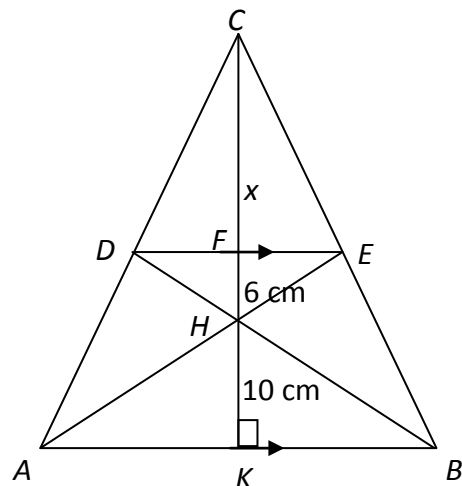
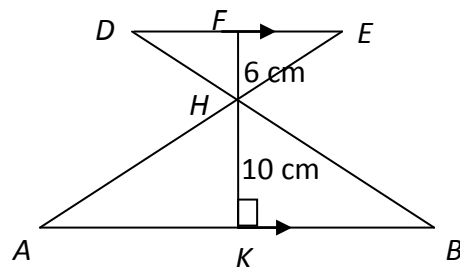
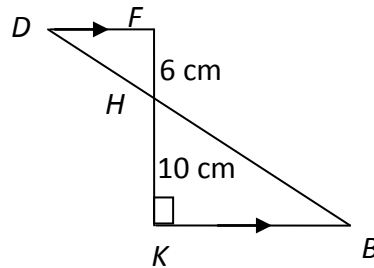
$$\frac{CD}{CA} = \frac{6}{10}$$

Triangle CDF is similar to triangle CAK .

$$\frac{CF}{CK} = \frac{CD}{CA}$$

$$\frac{x}{x + 16} = \frac{6}{10}$$

$$x = 24$$



Module 3: Graphs and relations**Question 1**

For the straight line with equation $6y - 5x = 30$, which of the following statements is true?

- A. the y-intercept is 30
- B. the gradient is -5
- C. **the x-intercept is -6**
- D. as x increases, y decreases
- E. the straight line has a negative slope

Answer is C

Worked Solution

Let's examine all five options.

Option A: Let's evaluate the y-intercept of this straight line equation by making y the subject.

$$\begin{aligned} 6y - 5x &= 30 \\ 6y &= 5x + 30 \\ y &= \frac{5}{6}x + \frac{30}{6} \\ y &= \frac{5}{6}x + 5 \end{aligned}$$

By substituting $x = 0$ into this equation, we find the y-intercept to be 5, not 30.

So option A is not the right answer.

Option B: The gradient is $\frac{5}{6}$, not -5 . So option B is not the right answer.

Option C: Let's evaluate the x-intercept by substituting 0 for the y value.

$$\begin{aligned} 6 \times 0 - 5x &= 30 \\ -5x &= 30 \\ x &= -6 \end{aligned}$$

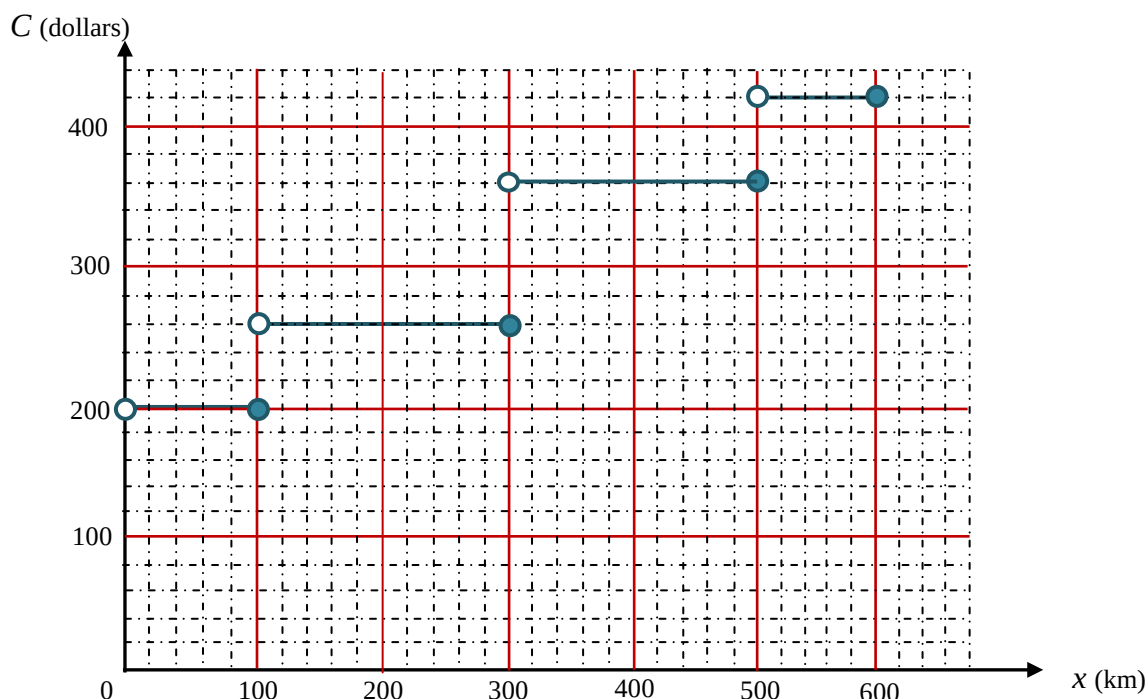
So option C is the right option

Option D: Because the gradient is positive, as x increases y also increases. So option D is not the right option.

Option E: The slope is positive, so option E is not the right answer.

Use the following information to answer Questions 2 and 3.

The charge, C , in dollars, of a car rental company is based on the number of kilometres, x , the car travelled during the rental period. The graph below shows the charge of renting a car depending on its usage, in kilometres.



Question 2

Jacqueline rents a car from the car rental company and travels for a certain distance using this car. She then returns this car to the company and rents another car.

She travels a total of 600 kilometres with the two cars.

The minimum amount of money that she would pay for renting the two cars is

- A. \$420
- B. \$520
- C. \$560
- D. \$620
- E. \$780

Answer is B

Worked Solution

If Jacqueline drove each car for 300 km, she would pay a total of $\$260 + \$260 = \$520$. To rent two cars and drive them each 300 km, this is the cheapest rental charge possible.



Tip

An answer of A would have been obtained if it had been assumed that only one car was rented, instead of two cars, and that the one rental car was driven for 600 km.

Question 3

Jacqueline's friend Sally rented three cars for her family and friends during her vacation in Queensland. She paid a total of \$920.

The usage of each car could have been

- A. 150 km, 200 km and 300 km, respectively.
- B. 300 km, 400 km and 500 km, respectively.
- C. 120 km, 280 km and 190 km, respectively.
- D. 320 km, 500 km and 100 km, respectively.**
- E. 350 km, 490 km and 220 km, respectively.

Answer is D

Worked Solution

Let's determine the car rental charge for all five options.

- Option A:** The car rental charge for 150 km, 200 km and 300 km are \$260 each. Total charge is \$780. So option A is not the right answer.
- Option B:** The car rental charge for 300 km is \$260, 400 km is \$360 and 500 km is \$360. Total charge is \$980. So option B is not the right answer.
- Option C:** The car rental charge for 120 km, 280 km and 190 km are \$260 each. Total charge is \$780. So option C is not the right answer.
- Option D:** The car rental charge for 320 km is \$360, 500 km is \$360 and 100 km is \$200. Total charge is \$920. So option D is the right answer.
- Option E:** The car rental charge for 350 km is \$360, 490 km is \$360 and 220 km is \$260. Total charge is \$980. So option E is not the right answer.

Question 4

The line joining the points (2, 2) and (−4, −13) would have the equation

- A. $y = \frac{5}{2}x - 3$
- B. $5x + 2y = 6$
- C. $y = \frac{7}{2}x - 5$
- D. $7x - 2y = 5$
- E. $2x + 2y = -13$

Answer is A

Worked Solution

$$\text{the gradient} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-13 - 2}{-4 - 2} = \frac{-15}{-6} = \frac{5}{2}$$

Now let's substitute $m = \frac{5}{2}$ and the coordinates of the point (2, 2) into $y = mx + c$ to evaluate c .

$$\begin{aligned} 2 &= \frac{5}{2} \times 2 + c \\ c &= -3 \end{aligned}$$

So the correct equation is $y = \frac{5}{2}x - 3$.

Question 5

The system of simultaneous equations $5x + 3y = 9$ and $2y = 6 - \frac{10}{3}x$

- A. have a solution at point (3, 5).
- B. have a solution at point (1, -5).
- C. have a solution at point (-3, -7).
- D. have no solution.
- E. **have infinitely many solutions.**

Answer is E

Worked Solution

Let's start with rearranging the second equation.

$$2y = 6 - \frac{10}{3}x$$

$$6y = 18 - 10x$$

$$10x + 6y = 18$$

Case 1: For a pair of simultaneous equations to have a unique solution, the ratio of the coefficients of x variables must not be the same as the ratio of the coefficients of y variables. Let's examine the equations.

$$5x + 3y = 9 \quad \text{and} \quad 10x + 6y = 18$$

$$\frac{5}{10} = \frac{3}{6} = \frac{1}{2}$$

So, this pair of simultaneous equations does not have a unique solution

A, B and C cannot be the right options

Case 2: If the ratio of the coefficients of x variables and the ratio of the coefficients of y variables are equal but the ratio of the constants are different, then the pair of simultaneous equations have no solution.

Case 3: If the ratio of the coefficients of x variables, the ratio of the coefficients of y variables and the ratio of the constants are all equal, then the pair of simultaneous equations have infinitely many solutions.

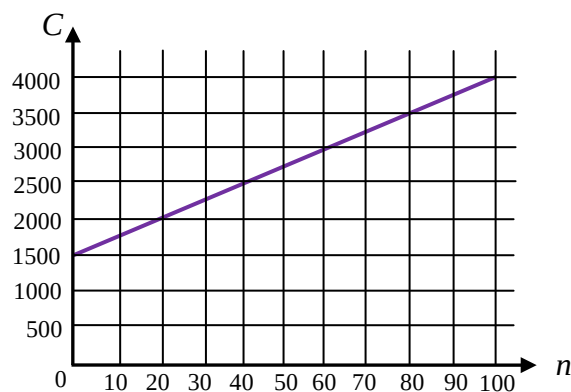
Let's examine the equations.

$$5x + 3y = 9 \quad \text{and} \quad 10x + 6y = 18$$

$$\frac{5}{10} = \frac{3}{6} = \frac{9}{18} = \frac{1}{2}$$

So, this pair of simultaneous equations has infinitely many solutions

Use the following information to answer Questions 6 and 7.



On the graph above, the straight line representing the cost of the production of n jackets by a textile company is given.

The revenue equation for the sale of n jackets is $R = 50n$.

Question 6

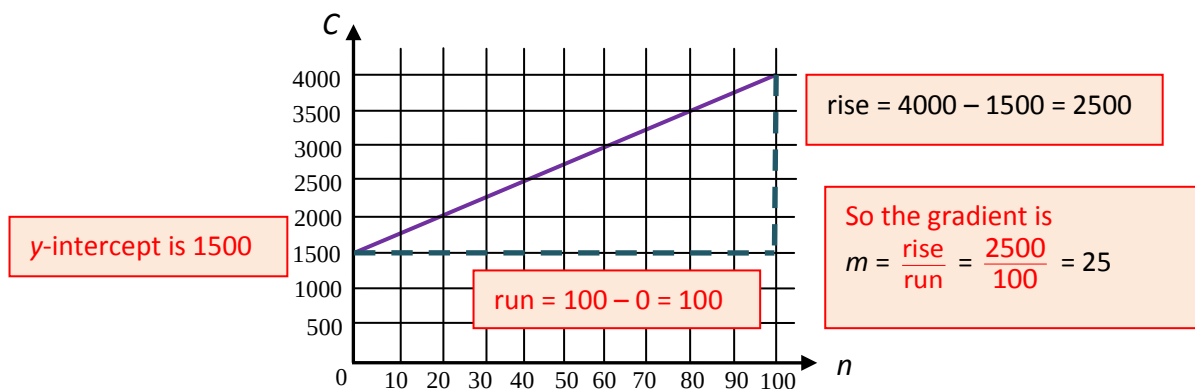
The number of jackets needed to be produced, and sold, to break even is

- A. 30
- B. 45
- C. **60**
- D. 75
- E. 90

Answer is C

Worked Solution

At break-even point, revenue and cost equations are equal to each other. Let's determine the cost equation using the straight line graph given to us.



So the cost equation is $C = 25n + 1500$.

At the break-even point, revenue equals cost.

$$R = C$$

$$50n = 25n + 1500$$

$$n = 60$$

So the number of jackets needed to be produced and sold to break even is 60.

Question 7

A particular number of jackets was produced and sold. It resulted in a profit of \$3000. The number of jackets produced and sold was

- A. 60
- B. 90
- C. 100
- D. 120
- E. **180**

Answer is E

Worked Solution

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$3000 = 50n - (25n + 1500)$$

$$3000 = 50n - 25n - 1500$$

$$3000 + 1500 = 25n$$

$$4500 = 25n$$

$$n = 180$$

180 jackets were produced and sold in order to obtain a profit of \$3000.

**Tip**

- An answer of A would have been obtained if the bracket for the cost equation was forgotten. Students generally make this bracket error while finding the profit. The wrong calculation is done as follows:

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$3000 = 50n - 25n + 1500$$

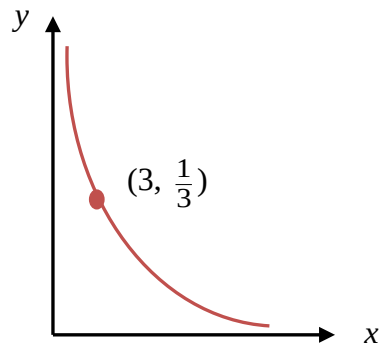
$$3000 - 1500 = 25n$$

$$1500 = 25n$$

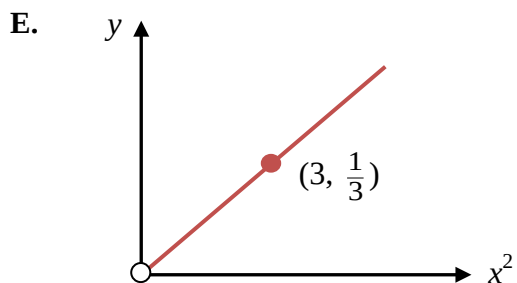
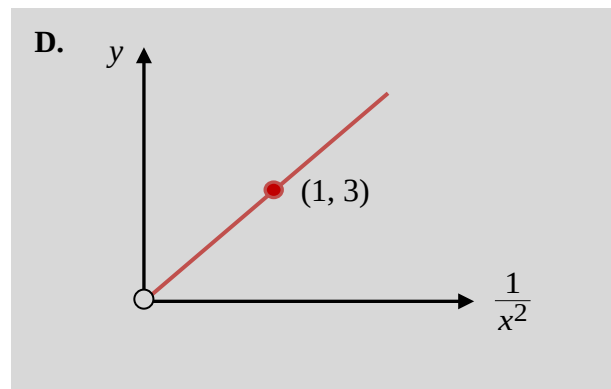
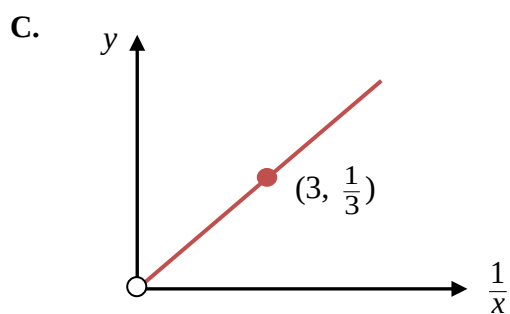
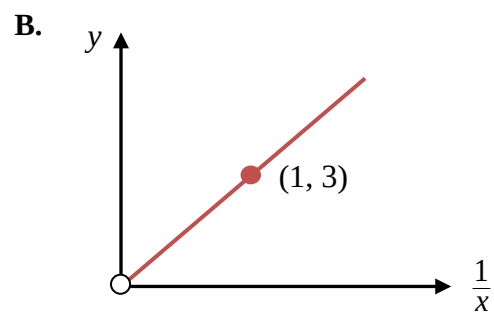
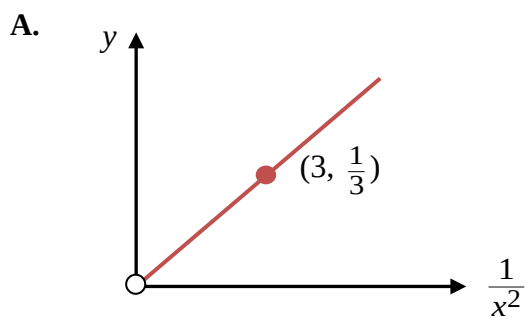
$$n = 60$$

Question 8

A graph representing the relation between y and x is shown below.



A graph that shows this same relationship could be



Answer is D

Worked Solution

The equation is in the form $y = \frac{k}{x}$ or $y = \frac{k}{x^2}$

Case 1: If the equation is in the $y = \frac{k}{x}$ form, we can evaluate k by substituting the point

$(3, \frac{1}{3})$ into the equation

$$\frac{1}{3} = \frac{k}{3} \rightarrow k = 1$$

Option B is a y versus $\frac{1}{x}$ graph where the gradient is $k = \frac{3}{1} = 3 \neq 1$.

So option B cannot be the answer.

Option C is a y versus $\frac{1}{x}$ graph where the gradient is $k = \frac{\frac{1}{3}}{1} = \frac{1}{3} \neq 1$.

So option C cannot be the answer.

Case 2: If the equation is in the $y = \frac{k}{x^2}$ form, we can evaluate k by substituting the point

$(3, \frac{1}{3})$ into the equation.

$$\frac{1}{3} = \frac{k}{3^2} \rightarrow k = 3$$

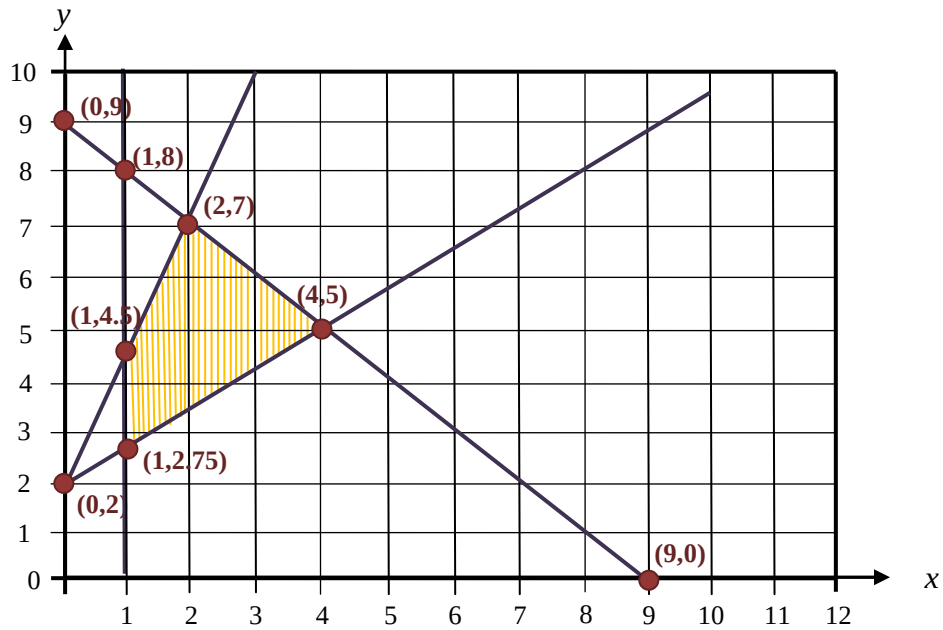
Option A is a y versus $\frac{1}{x^2}$ graph where the gradient is $k = \frac{\frac{1}{3}}{1} = \frac{1}{3} \neq 3$.

So option A cannot be the answer.

Option D is a y versus $\frac{1}{x^2}$ graph where the gradient is $k = \frac{3}{1} = 3$.

So option D can be the answer.

Question 9



In the graph above, the shaded region, with boundary included, represents the feasible region for a linear programming problem. The objective function for this problem is Z .

The minimum value of Z occurs at any point that lies on the line that joins points $(2, 7)$ and $(4, 5)$ in the feasible region.

The rule for the objective function, Z , could be

- A. $Z = x + y$
- B. $Z = 2x + 3y$
- C. $Z = 3x - 2y$
- D. $Z = -2x - 2y$
- E. $Z = x - y$

Answer is D**Worked Solution**

Let's examine the options by substituting the four vertices of the feasible region into the objective function, Z .

The vertices of the feasible region are $(1, 2.75)$, $(1, 4.5)$, $(2, 7)$ and $(4, 5)$.

Option A: $Z = x + y$
 $(1, 2.75) \rightarrow Z = 1 + 2.75 = 3.75$
 $(1, 4.5) \rightarrow Z = 1 + 4.5 = 5.5$
 $(2, 7) \rightarrow Z = 2 + 7 = 9$
 $(4, 5) \rightarrow Z = 4 + 5 = 9$

The minimum value of this objective function for the given feasible region occurs at the point $(1, 2.75)$ and it is 3.75. So option A is not the answer.

Option B: $Z = 2x + 3y$
 $(1, 2.75) \rightarrow Z = 2 \times 1 + 3 \times 2.75 = 10.25$
 $(1, 4.5) \rightarrow Z = 2 \times 1 + 3 \times 4.5 = 15.5$
 $(2, 7) \rightarrow Z = 2 \times 2 + 3 \times 7 = 25$
 $(4, 5) \rightarrow Z = 2 \times 4 + 3 \times 5 = 23$

The minimum value of this objective function for the given feasible region occurs at the point $(1, 2.75)$ and it is 10.25. So option B is not the answer.

Option C: $Z = 3x - 2y$
 $(1, 2.75) \rightarrow Z = 3 \times 1 - 2 \times 2.75 = -2.5$
 $(1, 4.5) \rightarrow Z = 3 \times 1 - 2 \times 4.5 = -6$
 $(2, 7) \rightarrow Z = 3 \times 2 - 2 \times 7 = -8$
 $(4, 5) \rightarrow Z = 3 \times 4 - 2 \times 5 = 2$

The minimum value of this objective function for the given feasible region occurs at the point $(2, 7)$ and it is -8 . So option C is not the answer.

Option D: $Z = -2x - 2y$
 $(1, 2.75) \rightarrow Z = -2 \times 1 - 2 \times 2.75 = -7.5$
 $(1, 4.5) \rightarrow Z = -2 \times 1 - 2 \times 4.5 = -11$
 $(2, 7) \rightarrow Z = -2 \times 2 - 2 \times 7 = -18$
 $(4, 5) \rightarrow Z = -2 \times 4 - 2 \times 5 = -18$

The minimum value of this objective function for the given feasible region occurs at the points $(2, 7)$ and $(4, 5)$ and it is -18 .

This means that the minimum value of Z occurs at any point that lies on the line that joins points $(2, 7)$ and $(4, 5)$ in the feasible region. So option D is the right answer.

Option E: $Z = x - y$
 $(1, 2.75) \rightarrow Z = 1 - 2.75 = -1.75$
 $(1, 4.5) \rightarrow Z = 1 - 4.5 = -3.5$
 $(2, 7) \rightarrow Z = 2 - 7 = -5$
 $(4, 5) \rightarrow Z = 4 - 5 = -1$

The minimum value of this objective function for the given feasible region occurs at the point $(2, 7)$ and it is -5 . So option E is not the answer.

Module 4: Business-related mathematics**Question 1**

Alice received \$460 750 from the sale of her house after paying the real estate agent's commission.

The real estate agent deducted a commission of 3% of the sale price before giving the balance to Alice. The sale price of Alice's house was

- A. \$475 000
- B. \$490 000
- C. \$446 927.50
- D. \$474 572.50
- E. \$520 000

Answer is A

Worked Solution

We need to evaluate the selling price of Alice's house = x .

$$x - x \times \frac{3}{100} = 460\,750$$

$$\frac{97}{100}x = 460\,750$$

$$97x = 460\,750 \times 100$$

$$x = \frac{46\,075\,000}{97}$$

$$x = 475\,000$$

The sale price of Alice's house was \$ 475 000.

Question 2

The amount of interest earned on a principal of \$120 000 invested at 5.6% per annum for five years, compounding quarterly would be given by

- A. $120\,000 \times 1.56^{20}$
- B. $120\,000 \times 1.014^{20}$
- C. $120\,000(1.56^{20} - 1)$
- D. $120\,000 \times 1.56^5$
- E. $120\,000(1.014^{20} - 1)$

Answer is E

Worked Solution

$$R = 1 + \frac{5.6}{100} = 1.014 \quad \text{and} \quad n = 4 \times 5 = 20$$

$$A = PR^n$$

$$A = 120\,000 \times 1.014^{20}$$

$$I = A - P$$

$$I = 120\,000 \times 1.014^{20} - 120\,000$$

$$I = 120\,000(1.014^{20} - 1)$$

Question 3

Jordan signs a hire-purchase contract to buy a new sofa set.
 The contract specifies that there are to be monthly payments for eighteen months.
 The flat rate of interest is 9.3% per annum.

The effective interest rate for this contract is closest to

- A. 21.5%
- B. 18.6%
- C. **17.6%**
- D. 15.8%
- E. 14.8%

Answer is C

Worked Solution

$$\text{effective rate} = \text{flat rate} \times \frac{2n}{n+1}$$

$$\text{effective rate} = 9.3 \times \frac{2 \times 18}{18 + 1}$$

$$\text{effective rate} = 17.621\% \cong 17.6\%$$

Question 4

Janna bought a box of chocolates for \$18 in January 2010.
 The inflation rate was 2.8% in 2010 and 3.5% in 2011.

How much would Janna pay for the same box of chocolates at the start of 2012?

- A. \$18.50
- B. **\$19.15**
- C. \$19.82
- D. \$23.04
- E. \$31.10

Answer is B

Worked Solution

$$\text{For 2010, } r = 1 + \frac{2.8}{100} = 1.028$$

$$\$18 \times 1.028 = \$18.504$$

So, the box of chocolates costs \$18.504 at the start of 2011.

$$\text{For 2011, } r = 1 + \frac{3.5}{100} = 1.035$$

$$\$18.504 \times 1.035 = \$19.15164 \cong \$19.15$$

So, Janna would pay \$19.15 for the box of chocolates at the start of 2012.

Question 5

A school purchased a photocopier for \$24 500 and it depreciates at a flat rate of 22.5% per annum.

If the photocopier's scrap value is \$3000, the number of years before the photocopier will be written off is closest to

- A. 2 years
- B. 3 years
- C. **4 years**
- D. 5 years
- E. 6 years

Answer is C

Worked Solution

$$\text{number of years} = \frac{\text{cost price} - \text{book value}}{\text{annual depreciation amount}}$$

$$\text{number of years} = \frac{\$24\,500 - \$3000}{\$24\,500 \times 0.225}$$

$$\text{number of years} = 3.9002 \cong 4 \text{ years}$$

Question 6

Ellen wants to invest \$100 000 at 5% per annum, compounding monthly, for 5 years.

Another option the bank provides for her is to invest her money at a flat rate of interest of 6% per annum for 5 years.

Ellen calculates the interest that she will receive for each option and finds out that she will be better off with the second option.

According to her calculations, the difference in the interest that she will receive from each of the two options at the end of the 5-year period is closest to

- A. \$28 335.87
- B. \$3822.56
- C. \$2371.84
- D. \$5000
- E. **\$1664.13**

Answer is E

Worked Solution

First we'll calculate the amount of compound interest using the *TVM solver* on a calculator.

The screenshot shows a calculator interface with a window titled "Edit Calculations". Below the title bar is a toolbar with various icons. The main area is titled "Compound Interest" and contains the following fields:

N	60
I%	5
PV	-100000
PMT	0
FV	128335.8679
P/Y	12
C/Y	12

At the bottom of the window are two buttons: "Help" and "Format".

Interest amount = \$128 335.8679 – \$100 000 = \$28 335.8679

Now we will calculate the amount of the flat rate of interest.

$$I = \frac{Prt}{100}$$

$$I = \frac{\$100\,000 \times 6 \times 5}{100}$$

$$I = \$30\,000$$

the difference = \$30 000 – \$28 335.8679

the difference = \$1664.1321 $\approx$ \$1664.13

Question 7

Jessica borrows \$85 000 and makes weekly repayments of \$210 at an interest of 8.4% per annum.

Two years after she takes out the loan, the interest rate decreases to 6.2% per annum.

If she continues to repay the same weekly amount after the rate decrease, the reduction in the time it will take her to repay the whole loan will be closest to

- A. 12.6 years
- B. 10.6 years
- C. 4.5 years
- D. **1.4 years**
- E. 1.1 years

Answer is D

Worked Solution

First we'll calculate the time it takes to pay off \$85 000 at an interest rate of 8.4% using the *TVM solver* on a calculator.

The screenshot shows a calculator window titled "Edit Calculations" with a "Compound Interest" section. The fields are as follows:

Field	Value
N	657.2605547
I%	8.4
PV	85000
PMT	-210
FV	0
P/Y	52
C/Y	52

Buttons for "Help" and "Format" are visible at the bottom.

So it takes 657.26 weeks to pay off \$85 000 at an interest rate of 8.4%.

Now we will calculate the amount Jessica still owes the bank after 2 years.

The screenshot shows the same calculator window with the following values:

Field	Value
N	104
I%	8.4
PV	85000
PMT	-210
FV	-76775.06746
P/Y	52
C/Y	52

Buttons for "Help" and "Format" are visible at the bottom.

So she owes the bank a total of \$76 775.06746 after two years.

Now we will calculate the time it takes to pay off the remaining loan at an interest rate of 6.2%.

Question 9

Jeremy and Jane borrow \$456 000 to purchase their home.

They are offered a reducing balance loan for 30 years with an interest rate of 8.65% per annum, compounding fortnightly.

They decide to repay the loan in 15 years instead of 30 years and there are no penalties for repaying the loan early.

Which of the following statements is **not** true?

- A. The total amount of repayments will be less.
- B. Each fortnightly repayment will increase.
- C. The amount they will save will be more than \$450 000.
- D. Each fortnightly repayment will be more than \$2000.
- E. **The amount they owe after 10 years will be less than \$200 000.**

Answer is E

Worked Solution

Let's do some calculations by using the *TVM solver* on a calculator.

The image shows two side-by-side screenshots of a TVM solver interface. Both windows are titled 'Edit Calculations' and have a 'Compound Interest' section. The left window shows calculations for a 30-year loan (N=780), with a PMT of -1640.026564. The right window shows calculations for a 15-year loan (N=390), with a PMT of -2089.070747. Both windows have the same interest rate (I%=8.65), present value (PV=456000), future value (FV=0), payments per year (P/Y=26), and compounding periods per year (C/Y=26). At the bottom of each window are 'Help' and 'Format' buttons.

Parameter	30-year Loan (N=780)	15-year Loan (N=390)
N	780	390
I%	8.65	8.65
PV	456000	456000
PMT	-1640.026564	-2089.070747
FV	0	0
P/Y	26	26
C/Y	26	26

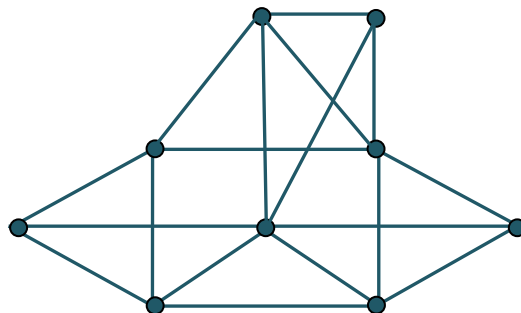
total amount of repayments when the loan is repaid in 30 years = $1640.026\ 564 \times 780$
 = \$1 279 220.719 92 $\approx$ \$1 279 220.72

total amount of repayments when the loan is repaid in 15 years = $2089.070\ 747 \times 390$
 = \$814 737.591 33 $\approx$ \$814 737.59

the difference = $\$1\ 279\ 220.72 - \$814\ 737.59 = \$464\ 483.13$

The total amount of repayments will be \$464 483.13 less. So options A and C are both correct.

Each fortnightly repayment increases from \$1640 to \$2089. So option B is also correct.

Module 5: Networks and decision mathematics**Question 1**

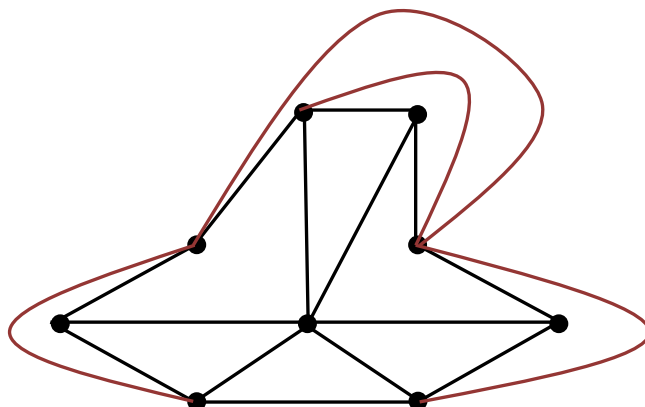
The graph shown above has

- A. 9 faces
- B. 11 faces**
- C. 14 faces
- D. 18 faces
- E. 19 faces

Answer is B

Worked Solution

The graph can be redrawn as a planar graph so that no edges cross.



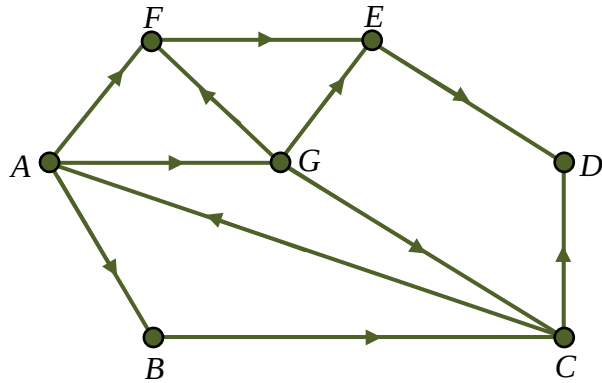
Now the graph has become planar and Euler's formula can be applied. There are 9 vertices and 18 edges.

$$v + f = e + 2$$

$$9 + f = 18 + 2$$

$$f = 11$$

Question 2



If matrix R is the adjacency matrix for the network above, 'vertex G is reachable from vertex B ' is shown by

- A. R, R^2 and R^5 .
- B. R^2 and R^3 .
- C. R^3 and R^6 .
- D. R^2, R^4 and R^7 .
- E. R^2, R^3 and R^6 .

Answer is C

Worked Solution

Let's construct the adjacency matrix, R , and evaluate R^2 , R^3 , R^4 , R^5 , R^6 and R^7 using the calculator.

$$R = \begin{matrix} & \begin{matrix} A & B & C & D & E & F & G \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$R^2 = \begin{matrix} & \begin{matrix} A & B & C & D & E & F & G \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 0 & 0 & 2 & 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$R^3 = \begin{matrix} & \begin{matrix} A & B & C & D & E & F & G \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 2 & 0 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

$$R^4 = \begin{matrix} & \begin{matrix} A & B & C & D & E & F & G \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 0 & 2 & 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 2 & 0 & 2 & 1 & 0 \\ 2 & 0 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 2 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$R^5 = \begin{matrix} & \begin{matrix} A & B & C & D & E & F & G \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 0 & 0 & 4 & 0 & 4 & 2 & 0 \\ 2 & 0 & 0 & 4 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 4 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$R^6 = \begin{matrix} & \begin{matrix} A & B & C & D & E & F & G \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 4 & 0 & 0 & 8 & 2 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 4 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 2 & 2 \end{bmatrix} \end{matrix}$$

$$R^7 = \begin{matrix} & \begin{matrix} A & B & C & D & E & F & G \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 0 & 4 & 0 & 2 & 0 & 4 & 4 \\ 0 & 0 & 4 & 0 & 4 & 2 & 0 \\ 4 & 0 & 0 & 8 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 4 & 2 & 0 \end{bmatrix} \end{matrix}$$

R , R^2 , R^4 , R^5 and R^7 all have a 0 in the position connecting B and G in the matrix, indicating that there is no path between B and G . (One-, two-, four-, five- and seven-step paths do not exist.)

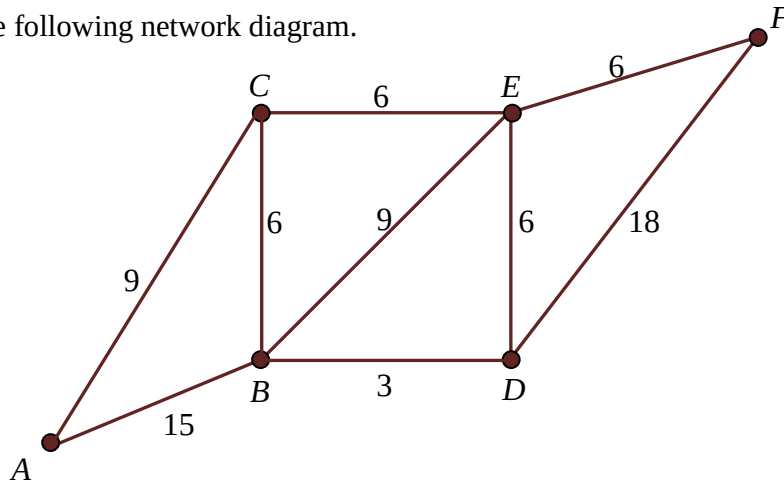
R^3 has a 1 in the position, indicating there is a three-step path between B and G .

R^6 has a 2 in the position, indicating there are two six-step paths between B and G .

So 'vertex G is reachable from vertex B ' is shown by R^3 and R^6 .

Use the following information to answer Questions 3, 4 and 5.

Consider the following network diagram.



Question 3

The degree of vertex E is

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6

Answer is C

Worked Solution

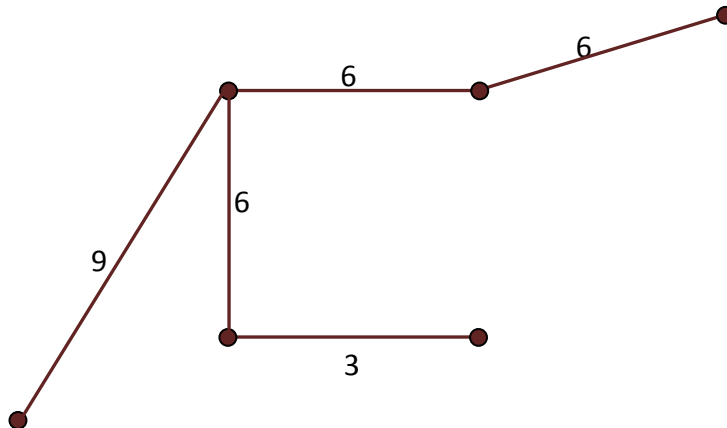
The degree of vertex E is the number of edges joined to it. Since four edges join to vertex E , its degree is 4.

Question 4

The minimum spanning tree for the network above has a weight of

- A. 78
- B. 60
- C. 33
- D. 30
- E. 27

Answer is D

Worked Solution

weight of the minimum spanning tree = $9 + 6 + 6 + 6 + 3 = 30$

Question 5

The shortest path from A to E is

- A. $A \rightarrow B \rightarrow E$
- B. $A \rightarrow C \rightarrow E$
- C. $A \rightarrow B \rightarrow D \rightarrow E$
- D. $A \rightarrow B \rightarrow C \rightarrow E$
- E. $A \rightarrow C \rightarrow B \rightarrow D \rightarrow F \rightarrow E$

Answer is B

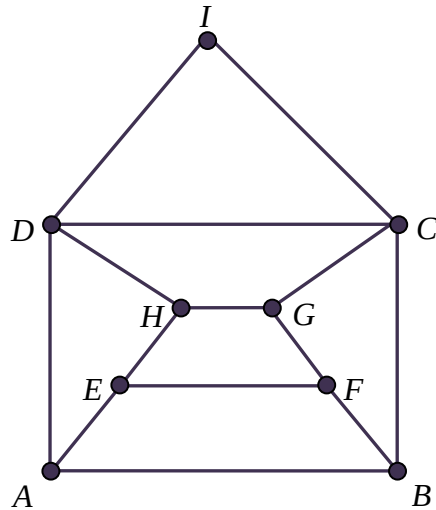
Worked Solution

Let's examine the options and find the length of each path in each of the five options.

- Option A:** $length = 15 + 9 = 24$
- Option B:** $length = 9 + 6 = 15$
- Option C:** $length = 15 + 3 + 6 = 24$
- Option D:** $length = 15 + 6 + 6 = 27$
- Option E:** $length = 9 + 6 + 3 + 18 + 6 = 42$

The shortest path from A to E is $A \rightarrow C \rightarrow E$ with a length of 15.

Question 6



A Hamiltonian circuit for the network above is

- A. *AEFB CGHDI*
- B. *IDA EHG FBC*
- C. *ABCDEF GHIA*
- D. *AEHGCIDHGFB*
- E. ***BADICGHEFB***

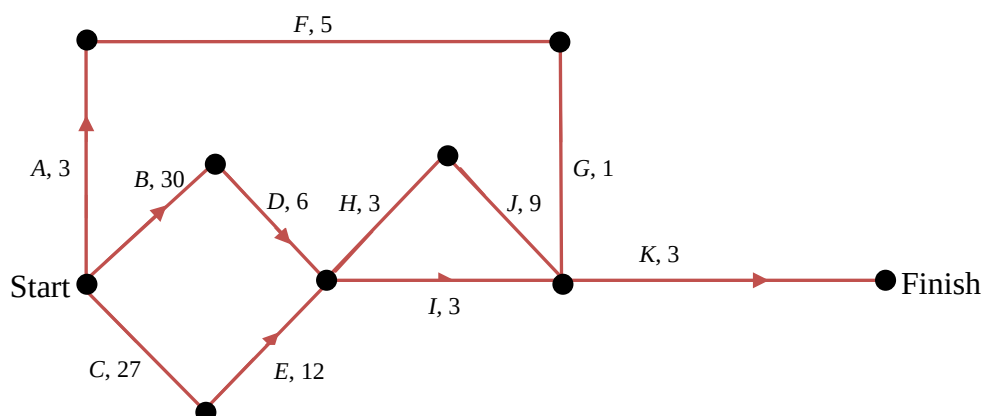
Answer is E

Worked Solution

BADICGHEFB is the only Hamiltonian circuit because it visits each vertex only once and it begins and ends with the same vertex.

Use the following information to answer Questions 7 and 8.

A project has 11 activities. The network below shows all these activities together with the time it takes, in days, to complete each activity.

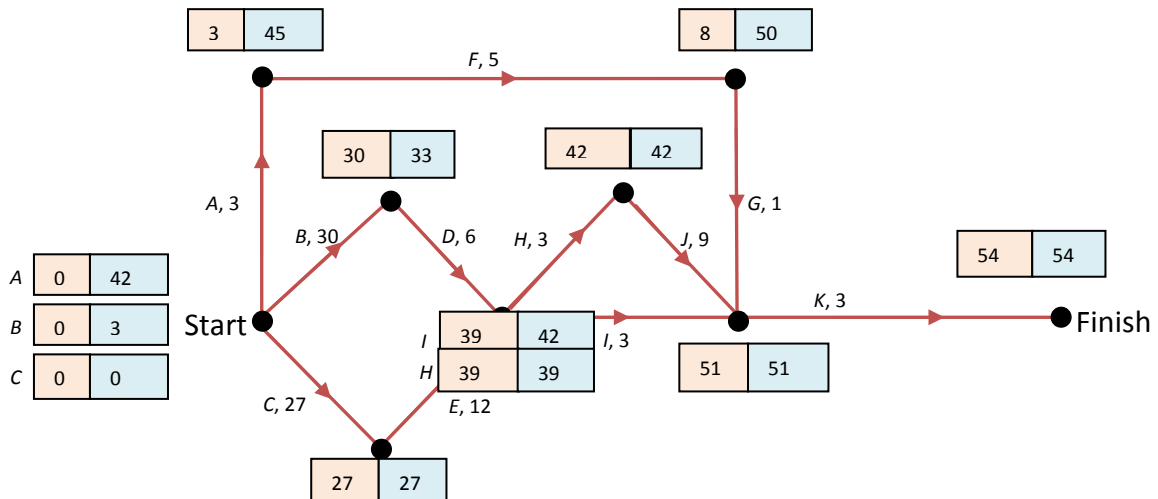


Question 7

The shortest time, in days, in which this project can be completed is

- A. 42
- B. 54
- C. 68
- D. 72
- E. 96

Answer is B

Worked Solution

In the boxes for the activities in the diagram above, the earliest start times are shown in the left-hand side of the box each time and the latest start times are shown in the right-hand side of the box. The critical path is CEHJK with a total length of 54 days.

So the shortest time in which this project can be completed is 54 days.

Question 8

Float time, in days, for activity *F* is

- A. 42
- B. 39
- C. 18
- D. 8
- E. 3

Answer is A

Worked Solution

Float time for activity *F* is $45 - 3 = 42$ days.

Question 9

The information below shows the time (in hours) that each of the three dance teachers, Reesha, Henry and Alexandra, would take to complete teaching each of the dance styles flamenco (*F*), Latin dance (*L*) and hip-hop (*H*) to the dance students for a school performance night.

	<i>F</i>	<i>L</i>	<i>H</i>
<i>Reesha</i>	32	44	51
<i>Henry</i>	41	52	53
<i>Alexandra</i>	28	25	27

If each teacher is allocated only one dance style to teach, the minimum total time, in hours, for this group of dance teachers to complete teaching all three dance styles for the performance night is

- A. 98
- B. 105
- C. **110**
- D. 117
- E. 125

Answer is C

Worked Solution

Step 1: Perform row reduction. Subtract the minimum element of each row from each element of that row. Then draw the minimum number of lines to cover all zeros.

$$\begin{bmatrix} 32 & 44 & 51 \\ 41 & 52 & 53 \\ 28 & 25 & 27 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 12 & 19 \\ 0 & 11 & 12 \\ 3 & 0 & 2 \end{bmatrix} \quad \text{Only two lines, so cannot start allocation.}$$

Step 2: Perform column reduction. Subtract the minimum element of each column from each element of that column. Then draw the minimum number of lines to cover all zeros.

$$\begin{bmatrix} 0 & 12 & 19 \\ 0 & 11 & 12 \\ 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 12 & 17 \\ 0 & 11 & 10 \\ 3 & 0 & 0 \end{bmatrix} \quad \text{Only two lines, so cannot start allocation.}$$

Step 3: Perform the Hungarian algorithm. Find the smallest uncovered number from step 2. Add this number to all covered numbers. At the intersections of straight lines, add this number twice. Then subtract the overall smallest number from all the numbers in the matrix.

$$\begin{bmatrix} 0 & 12 & 17 \\ 0 & 11 & 10 \\ 3 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 10 & 12 & 17 \\ 10 & 11 & 10 \\ 23 & 10 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 7 \\ 0 & 1 & 0 \\ 13 & 0 & 0 \end{bmatrix}$$

There are three lines, so we can now start our allocations.

	<i>F</i>	<i>L</i>	<i>H</i>
<i>Reesha</i>	0	2	7
<i>Henry</i>	0	1	0
<i>Alexandra</i>	13	0	0

The rows and columns with zeros are the ones indicating the allocation.

Alexandra should be allocated to Latin dance (she is the only one with a zero for Latin dance).

Henry and Alexandra can both be allocated to hip-hop. Since Alexandra is already allocated to Latin dance, **Henry must be allocated to hip-hop**.

Reesha and Henry could both be allocated to flamenco. Since Henry is allocated to hip-hop, **Reesha must be allocated to flamenco**.

the minimum total = 32 + 53 + 25 = 110 hours

Module 6: Matrices**Question 1**

The determinant of the matrix $\begin{bmatrix} -14 & 2a \\ 28 & -3 \end{bmatrix}$ is 154.

The value of a is

- A. 4
- B. -4
- C. 2
- D. -2
- E. 7

Answer is D

Worked Solution

$$\begin{aligned}
 \text{the determinant:} \quad & -14 \times -3 - 28 \times 2a = 154 \\
 & 42 - 56a = 154 \\
 & -56a = 154 - 42 \\
 & -56a = 112 \\
 & a = -2
 \end{aligned}$$

Question 2

Given $\begin{bmatrix} -2 & 3 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 7 & 2 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix}$, the matrix calculation would be represented by

- A. $\begin{bmatrix} -2 + 7 \times 1 & 3 + 2 \times 5 \\ 2 + 1 \times 4 & 5 + 3 \times 2 \end{bmatrix}$
- B. $\begin{bmatrix} -2 + 7 \times 1 \times 2 \times 4 & 3 + 7 \times 5 \times 2 \times 2 \\ 2 + 1 \times 1 \times 3 \times 4 & 5 + 1 \times 5 \times 3 \times 2 \end{bmatrix}$
- C. $\begin{bmatrix} -2 + 7 + 1 & 3 + 2 + 5 \\ 2 + 1 + 4 & 5 + 3 + 2 \end{bmatrix}$
- D. $\begin{bmatrix} -2 \times 7 \times 1 & 3 \times 2 \times 5 \\ 2 \times 1 \times 4 & 5 \times 3 \times 2 \end{bmatrix}$
- E. $\begin{bmatrix} -2 + 7 \times 1 + 2 \times 4 & 3 + 7 \times 5 + 2 \times 2 \\ 2 + 1 \times 1 + 3 \times 4 & 5 + 1 \times 5 + 3 \times 2 \end{bmatrix}$

Answer is E

Worked Solution

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \text{ and } C = \begin{bmatrix} i & j \\ k & l \end{bmatrix}$$

$$\begin{aligned}
 \text{then } A + B \times C &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e \times i + f \times k & e \times j + f \times l \\ g \times i + h \times k & g \times j + h \times l \end{bmatrix} \\
 &= \begin{bmatrix} a + e \times i + f \times k & b + e \times j + f \times l \\ c + g \times i + h \times k & d + g \times j + h \times l \end{bmatrix}
 \end{aligned}$$

$$\text{So } \begin{bmatrix} -2 & 3 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 7 & 2 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -2 + 7 \times 1 + 2 \times 4 & 3 + 7 \times 5 + 2 \times 2 \\ 2 + 1 \times 1 + 3 \times 4 & 5 + 1 \times 5 + 3 \times 2 \end{bmatrix}$$

Question 3

If $A = \begin{bmatrix} -4 & 8 & 2 \\ 3 & 5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, then $3B - AC$ equals

- A. $\begin{bmatrix} 9 \\ -36 \end{bmatrix}$
- B. $\begin{bmatrix} -1 \\ -24 \end{bmatrix}$
- C. $\begin{bmatrix} 15 \\ -18 \end{bmatrix}$
- D. $\begin{bmatrix} 6 \\ 18 \end{bmatrix}$
- E. $\begin{bmatrix} -21 \\ 0 \end{bmatrix}$

Answer is A

Worked Solution

$$\begin{aligned} 3B - AC &= 3 \times \begin{bmatrix} 5 \\ -6 \end{bmatrix} - \begin{bmatrix} -4 & 8 & 2 \\ 3 & 5 & -1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ -18 \end{bmatrix} - \begin{bmatrix} 6 \\ 18 \end{bmatrix} \\ &= \begin{bmatrix} 9 \\ -36 \end{bmatrix} \end{aligned}$$

Question 4

For the matrix $\begin{bmatrix} -7 & 32 & -11 & -1 \\ 8 & 55 & 23 & -5 \\ 15 & 16 & 42 & 0 \\ 53 & -2 & 18 & 34 \\ 12 & -1 & 5 & 81 \end{bmatrix}$, the product of element $m_{4,2}$ and element $m_{3,2}$ is

- A. -115
- B. -46
- C. -32
- D. -80
- E. 0

Answer is C

Worked Solution

$$m_{4,2} \times m_{3,2} = -2 \times 16 = -32$$

**Tip**

➤ An answer of A would have been obtained if the product of element $m_{2,4}$ and element $m_{2,3}$ had been evaluated.

Question 5

The system of simultaneous equations

$$\begin{aligned} -2y + z + w &= -12 \\ x + y + z - w &= 21 \\ x + y + 2w &= 34 \\ x + y + z + w &= 48 \end{aligned}$$

can be solved by evaluating the matrix product

A.
$$\begin{bmatrix} 0 & -2 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} -12 \\ 21 \\ 34 \\ 48 \end{bmatrix}$$

B.
$$\begin{bmatrix} 0 & 2 & -1 & -1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & 0 & -2 \\ -1 & -1 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} -12 \\ 21 \\ 34 \\ 48 \end{bmatrix}$$

C.
$$\begin{bmatrix} 0 & -2 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 12 \\ -21 \\ -34 \\ -48 \end{bmatrix}$$

D.
$$\begin{bmatrix} 0.5 & 3 & 1.5 & -2 \\ -1 & -0.5 & -0.5 & 1 \\ 0 & -0.5 & -1 & 1 \\ 0 & -0.5 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -12 \\ 21 \\ 34 \\ 48 \end{bmatrix}$$

E.
$$\frac{1}{2} \begin{bmatrix} 1 & 3 & 3 & -4 \\ -1 & -1 & -1 & 2 \\ 0 & -1 & -2 & 3 \\ 0 & -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -12 \\ 21 \\ 34 \\ 48 \end{bmatrix}$$

Answer is E

Worked Solution

The matrix equation representing the system of simultaneous equations is

$$AX = C, \text{ where } A = \begin{bmatrix} 0 & -2 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} -12 \\ 21 \\ 34 \\ 48 \end{bmatrix}.$$

The solution of this matrix equation is

$$X = A^{-1} \times C$$

$$X = \begin{bmatrix} 0.5 & 1.5 & 1.5 & -2 \\ -0.5 & -0.5 & -0.5 & 1 \\ 0 & -0.5 & -1 & 1.5 \\ 0 & -0.5 & 0 & 0.5 \end{bmatrix} \times \begin{bmatrix} -12 \\ 21 \\ 34 \\ 48 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} 1 & 3 & 3 & -4 \\ -1 & -1 & -1 & 2 \\ 0 & -1 & -2 & 3 \\ 0 & -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -12 \\ 21 \\ 34 \\ 48 \end{bmatrix}$$

Question 6

The median house prices in towns A, B and C are to increase by 6%, 9% and 15%, respectively.

If current prices are, in matrix format, $H = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where a , b and c represent the current median

house prices in towns A, B and C, respectively, then the prices after the increase would be

A. $H \times I$, where $I = \begin{bmatrix} 0.06 & 0 & 0 \\ 0 & 0.09 & 0 \\ 0 & 0 & 0.15 \end{bmatrix}$

B. $H \times I$, where $I = \begin{bmatrix} 1.06 & 0 & 0 \\ 0 & 1.09 & 0 \\ 0 & 0 & 1.15 \end{bmatrix}$

C. $H \times I$, where $I = \begin{bmatrix} 6 \\ 9 \\ 15 \end{bmatrix}$

D. $H + I$, where $I = \begin{bmatrix} 1.06 \\ 1.09 \\ 1.15 \end{bmatrix}$

E. $H - I$, where $I = \begin{bmatrix} 1.06 \\ 1.09 \\ 1.15 \end{bmatrix}$

Answer is B

Worked Solution

We need to multiply H by I , so option D and E cannot be the right answers.

In order for us to do the multiplication, the matrix I needs to be a 3×3 matrix, so option C cannot be the right answer.

The matrix I should be in the form $I = \begin{bmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{bmatrix}$.

Since we need to increase the median house price for town A by 6%, $d = 1 + \frac{6}{100} = 1.06$.

Since we need to increase the median house price for town B by 9%, $e = 1 + \frac{9}{100} = 1.09$.

Since we need to increase the median house price for town C by 15%, $f = 1 + \frac{15}{100} = 1.15$.

Question 7

Matrix X is a 4×4 matrix and thirteen of the elements are zero.

Matrix Y contains twelve elements, none of which are zero.

Matrix Z contains six elements, two of which are zero.

Assuming the matrix product $(XY)Z$ is defined, the minimum number of zero elements in the product matrix $(XY)Z$ is

- A. 9
- B. 6
- C. 4
- D. 3
- E. 2

Answer is E

Worked Solution

Matrix Y has twelve elements and can be multiplied by matrix X . So matrix Y should be a 4×3 matrix.

Since thirteen of the elements in matrix X are zero, at least one row must be formed by all zero elements.

So matrix product XY must have at least one zero row. This means that matrix product XY must have at least three zero elements.

$$\text{e.g. } X \times Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Matrix Z has six elements and can be multiplied by matrix product XY , which is a 4×3 matrix. So matrix Z should be a 3×2 matrix.

Since matrix product XY is a 4×3 matrix and matrix Z is a 3×2 matrix, matrix product $(XY)Z$ must be a 4×2 matrix.

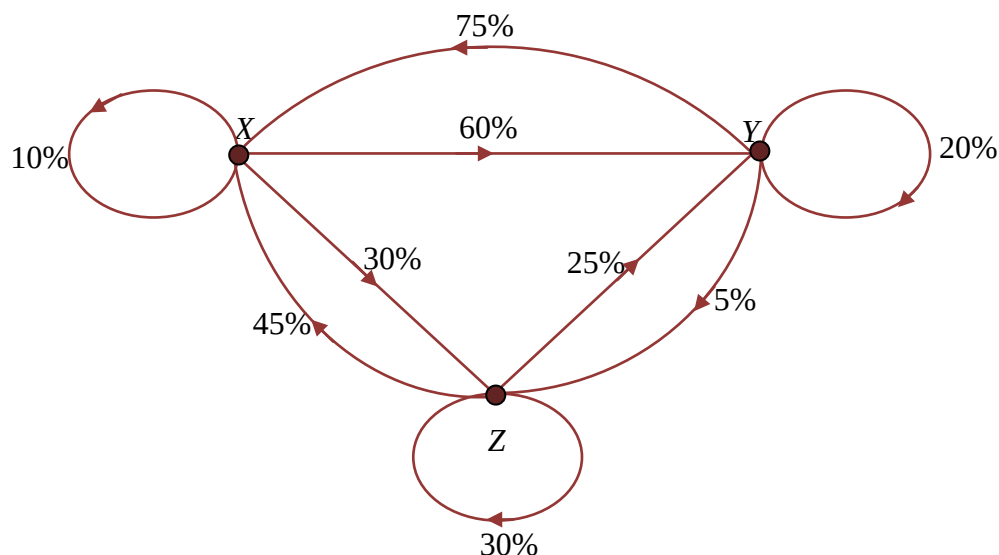
Since matrix product XY has a zero row, matrix product $(XY)Z$ must have a zero row as well. This means that matrix product $(XY)Z$ must have at least two zero elements.

$$\text{e.g. } (X \times Y)Z = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) \times \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 0 & 0 \end{bmatrix}$$

Use the following information to answer Questions 8 and 9.

Customers of a car service company can choose to take their vehicles to one of three car service branches: X, Y or Z.

The company had 2000 customers at the beginning. 1000 of those customers preferred to take their vehicles to branch X, 800 customers preferred branch Y and the rest preferred branch Z. The transition diagram below shows the percentage of customers who will take their vehicles to the same branch or to one of the other two branches for car services from month to month.



Question 8

Assume the pattern of movement shown in the transition diagram continues.

The total number of customers who will change the car service branch they take their vehicles to after one month is

- A. 320
- B. 450
- C. 1260
- D. **1680**
- E. 1850

Answer is D

Worked Solution

10% of customers will remain at branch X.

This means that $1000 \times \frac{10}{100} = 100$ customers will remain at branch X.

20% of customers will remain at branch Y.

This means that $800 \times \frac{20}{100} = 160$ customers will remain at branch Y.

30% of customers will remain at branch Z.

This means that $200 \times \frac{30}{100} = 60$ customers will remain at branch Z.

So a total of 320 customers will not change the car service branch they take their vehicles to after one month.

This means that $2000 - 320 = 1680$ people will change the car service branch they take their vehicles to after one month.

Question 9

The management board of the company made a decision to shut down the branch that has the least number of customers at the end of one year.

The number of customers the branch that will be shut down after one year has is closest to

- A. 400
- B. **411**
- C. 754
- D. 790
- E. 834

Answer is B

Worked Solution

Initial state matrix: $S_0 = \begin{bmatrix} 1000 \\ 800 \\ 200 \end{bmatrix}$

$$S_{12} = T^{12} \times S_0 = \begin{bmatrix} 0.10 & 0.75 & 0.45 \\ 0.60 & 0.20 & 0.25 \\ 0.30 & 0.05 & 0.30 \end{bmatrix} \times \begin{bmatrix} 1000 \\ 800 \\ 200 \end{bmatrix} = \begin{bmatrix} 834.3736553 \\ 754.2250141 \\ 411.4013306 \end{bmatrix} \cong \begin{bmatrix} 834 \\ 754 \\ 411 \end{bmatrix}$$

$\therefore$ Branch Z will have 411 customers after one year, so it will be closed.

END OF SOLUTIONS BOOK