



INSIGHT

YEAR 12 Trial Exam Paper

2012

FURTHER MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- correct solutions with full working
- explanatory notes
- mark allocations

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Core

Question 1

The school-assessed coursework results that 28 year 12 Further Mathematics students received from two different analysis tasks are given in the back-to-back stemplot shown in Figure 1 below.

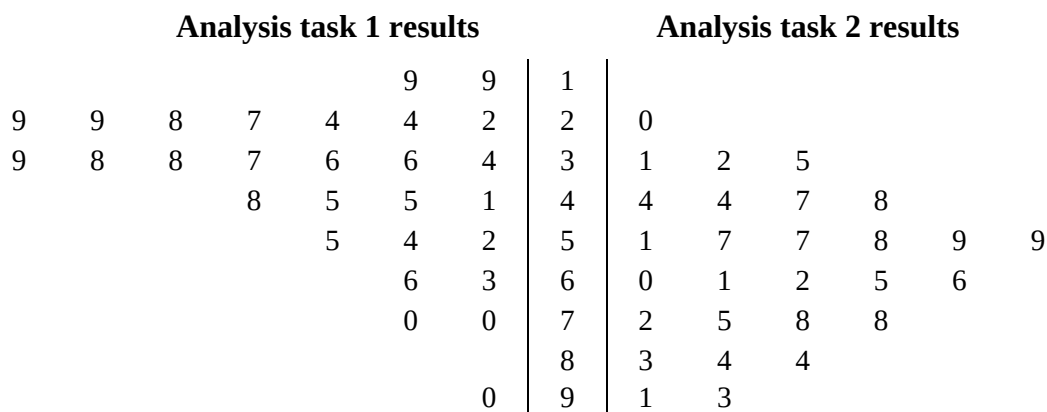
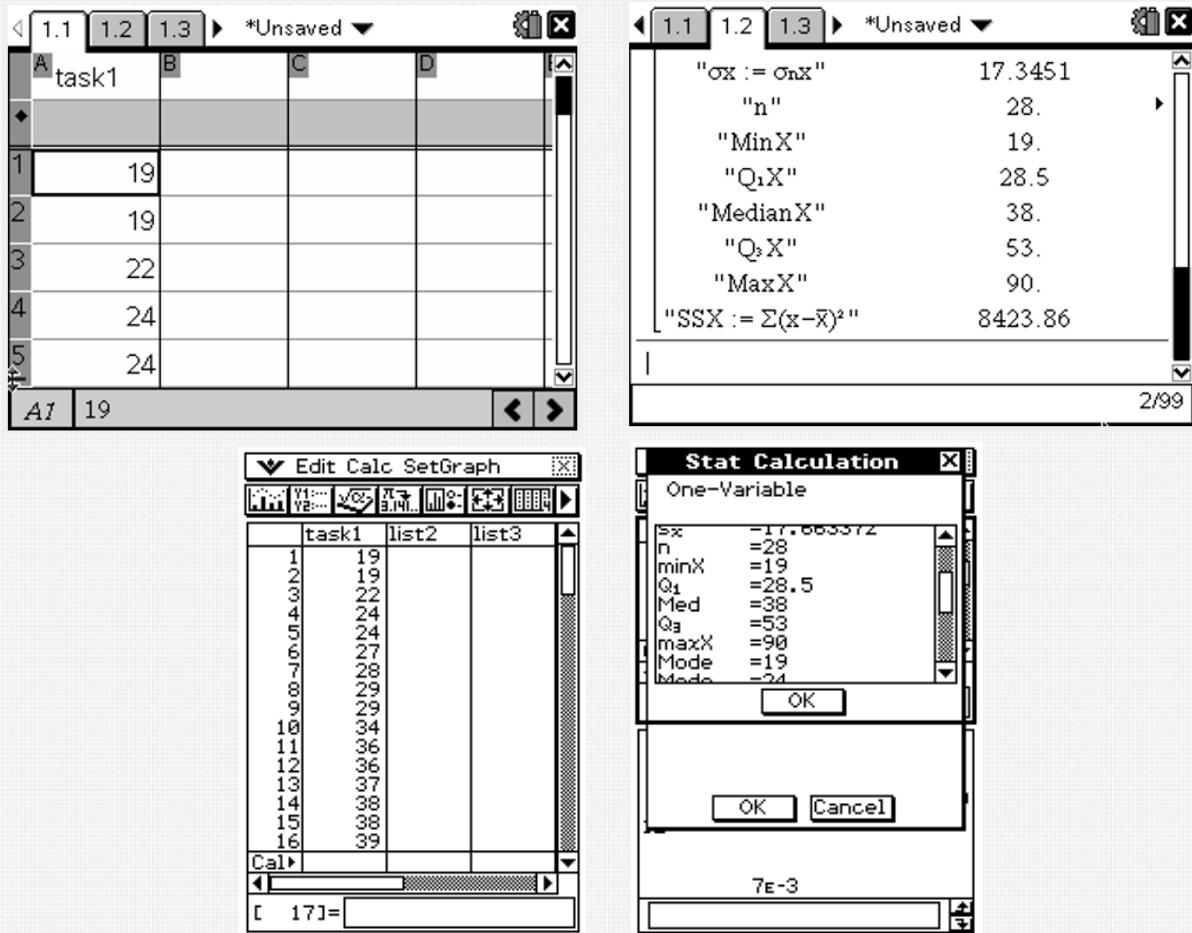


Figure 1

- a.** For these 28 students, determine
- i.** the median of the analysis task 1 results.

Worked solution

Let's calculate the five-figure summary of the data, by calculator.



As we can see from the calculation above, the median of the analysis task 1 results is 38.

1 mark

- ii. the interquartile range of the analysis task 1 results.

Worked solution

$$Q_1 = 28.5 \text{ and } Q_3 = 53$$

$$\text{So IQR} = Q_3 - Q_1 = 53 - 28.5 = 24.5$$

1 mark

1 + 1 = 2 marks

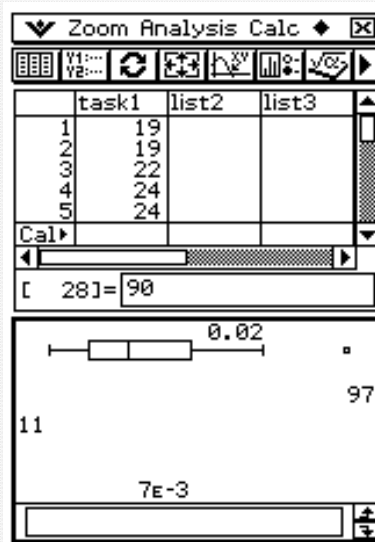
Core – continued
TURN OVER

- b. If the data values representing analysis task 1 results displayed in Figure 1 were used to construct a boxplot with outliers, then would the student who received 90 be shown as an outlier?

Show an appropriate calculation to support your answer.

Worked solution

First, let's draw a boxplot by calculator to see if 90 is an outlier or not.



As we can see on the boxplot above, the student who received 90 is an outlier.

Now let's calculate the upper fence to prove that the data value 90 is an outlier.

$$\text{Upper fence} = Q_3 + 1.5 \times \text{IQR} = 53 + 1.5 \times 24.5 = 89.75$$

As 90 is greater than 89.75, i.e. the upper fence, it is an outlier.

2 marks

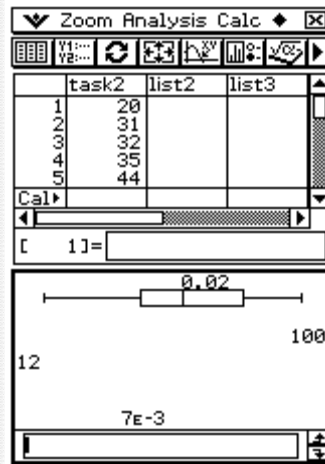
Mark allocation

- 1 mark for the correct calculation of the upper fence.
- 1 mark for stating that 90 is an outlier because it is greater than the upper fence.

- c. Both the median and the mean are appropriate measures of centre of the distribution of the analysis task 2 results.
Explain why.

Worked solution

Let's draw a boxplot to determine the shape of the distribution.



As we can see from the boxplot above, the distribution is approximately symmetrical. When the distribution is approximately symmetrical, both the median and the mean are appropriate measures of centre of the distribution.

1 mark

Question 2

The average monthly incomes (in dollars) and the average monthly expenditures (in dollars) for a group of 15 households have been recorded. The results are displayed in Table 1 below.

Average monthly income (\$)	Average monthly expenditure (\$)
3 000	2 900
1 500	2 500
5 100	3 200
5 200	4 700
3 400	2 500
7 500	6 000
10 250	11 000
2 500	2 350
4 200	4 000
6 500	5 800
8 250	7 250
12 300	10 750
4 200	4 100
2 700	2 000
8 900	9 200

Table 1

The aim is to find a linear equation that allows average monthly expenditure to be predicted from average monthly income.

- a. What will be the dependent variable in the equation?

Worked solution

The dependent variable in this equation will be *average monthly expenditure*.

1 mark

- b. The equation of the least squares regression is to be calculated for this set of data. It has been assumed therefore that the relationship between average monthly income and average monthly expenditure is _____.

Worked solution

In order to calculate the least squares regression equation, the relationship between average monthly income and average monthly expenditure must be linear.

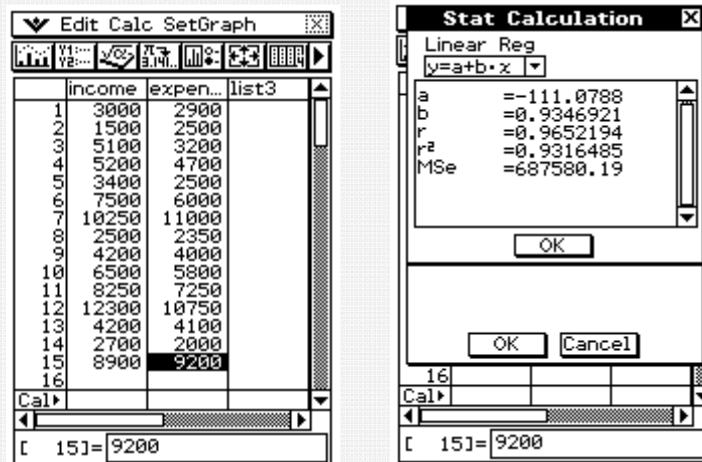
1 mark

- c. Determine the equation of the least squares regression line that enables *average monthly expenditure* to be predicted from *average monthly income*.
Write this equation in terms of the variables *average monthly expenditure* and *average monthly income*.
Give the coefficients, correct to two decimal places.

Worked solution

Independent variable: *average monthly income*

Dependent variable: *average monthly expenditure*



$$\text{average monthly expenditure} = -111.08 + 0.93 \times \text{average monthly income}$$

2 marks

Mark allocation

- 1 mark for calculating the gradient and the y intercept as 0.93 and -111.08.
- 1 mark for writing the equation of the least squares regression line.

- d. Using the equation that you have determined in **part c**, interpret the slope of the least squares regression line in terms of the variables *average monthly income* and *average monthly expenditure*.

Worked solution

On average, *average monthly expenditure* increases by \$0.93 for each \$1 increase in *average monthly income*.

1 mark

- e. Determine and interpret the coefficient of determination in terms of the variables *average monthly income* and *average monthly expenditure*.

Write your answer as a percentage, correct to two decimal places.

Worked solution

$$\text{coefficient of determination} = r^2 = 93.16\%$$

So 93.16% of the variation in *average monthly expenditure* can be explained by the variation in *average monthly income*.

1 mark

- f. i. Use the least squares regression line equation that you have determined in **part c** to predict the average monthly expenditure (in dollars) of a household for which the average monthly income is \$20 000.

Give your answer correct to the nearest hundred dollars.

Worked solution

$$\text{average monthly expenditure} = -111.08 + 0.93 \times \text{average monthly income}$$

$$\text{average monthly expenditure} = -111.08 + 0.93 \times \$20\,000$$

$$\text{average monthly expenditure} = \$18\,488.92 \approx \$18\,500$$

1 mark

- ii. The prediction made in **part f i** is not likely to be reliable.
Explain why.

Worked solution

Making the required prediction involves going beyond the data used (extrapolation) to determine the regression equation.

There was no evidence that the relationship continued for income levels above those given in the table. Therefore, the mathematical model established by the regression equation cannot be relied upon outside the range of the given data set.

1 mark

1 + 1 = 2 marks

Question 3

Table 2 shows the seasonal indices for ice-cream sales in summer, autumn, winter and spring.

Seasonal indices			
Summer	Autumn	Winter	Spring
1.525	0.755	0.625	1.095

Table 2

- a. By what percentage should the ice-cream sales figures for winter be increased in order to correct for seasonality?

Worked solution

Let's assume that 100 ice-creams are sold in winter.

$$\text{deseasonalised value} = \frac{\text{actual value}}{\text{seasonal index}}$$

$$\text{deseasonalised value} = \frac{100}{0.625} = 160$$

$$160\% - 100\% = 60\%$$

So the ice-cream sales figures for winter should be increased by 60% in order to correct for seasonality.

1 mark



Tip

- A common mistake that students make is to decrease the sales by 37.5%.
- A seasonal index of 0.625 means that the ice-cream sales in winter are 37.5% below the quarterly average. Since the winter ice-cream sales are lower than the quarterly average, we need to increase this value in order to correct for seasonality.

- b. What does a seasonal index of 1.525 tell us about the ice-cream sales in summer?

Worked solution

The seasonal index for ice-cream sales in summer is 1.525, which corresponds to 152.5%.

As the quarterly average is 100%, the ice-cream sales in spring are

$$152.5\% - 100\% = 52.5\% \text{ above the quarterly average.}$$

So the summer ice-cream sales are 52.5% above the average for the four seasons of the year.

1 mark

Total 15 marks

**END OF CORE
TURN OVER**

Module 1: Number patterns

Question 1

Susan received 34 out of 100 for her first English assignment in the 2012 academic year. She submitted all her assignments regularly and studied hard to improve her results. Consequently, she increased her mark by 6 in each of the subsequent assignments.

- a. What type of sequence is formed by Susan's assignment marks?

Worked solution

This is an arithmetic sequence with a common difference of 6.

1 mark

- b. What mark did Susan receive for her fifth English assignment?

Worked solution

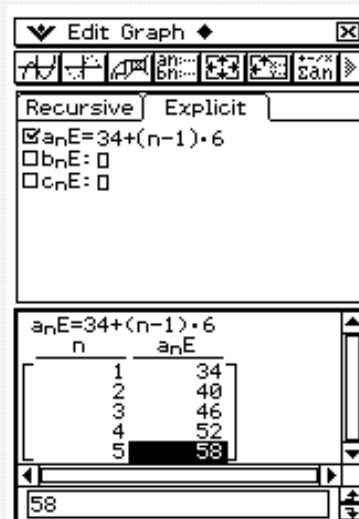
We can use the arithmetic sequence rule.

$$t_n = a + (n - 1)d \text{ where } t_1 = a = 34 \text{ and } d = 6$$

$$t_5 = 34 + 6 \times (5 - 1) = 34 + 6 \times 4 = 58$$

So Susan received 58 for her fifth assignment.

Alternatively, we can generate the sequence by calculator.



1 mark

- c. Susan received 100 for the last English assignment of the year.
How many English assignments did she have during 2012?

Worked solution

$$t_n = 100$$

$$34 + (n - 1)6 = 100$$

$$n = 12$$

Alternatively, we can generate the sequence by calculator.

n	a _n E
11	94
12	100
13	106
14	112
15	118

1 mark

- d. Find the average mark of Susan's English assignments for the year 2012.

Worked solution

Let's calculate the sum of Susan's twelve marks using the arithmetic sum formula.

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_{12} = \frac{12}{2}(2 \times 34 + (12 - 1)6)$$

$$S_{12} = 804$$

In order to find her average mark, we need to divide the sum of her twelve marks by 12.

$$\text{average assignment mark} = \frac{804}{12} = 67$$

So the average mark of Susan's English assignments for the year 2012 is 67.

1 mark

Question 2

When Susan's school first opened it had 240 students. The number of students enrolled at this school increased by 20% each subsequent year.

- a. Determine the number of students the school had
- i. after 1 year of opening.

Worked solution

This is a geometric sequence starting with 240.

$$\text{common ratio: } r = 1 + \frac{20}{100} = 1.2$$

$$\text{Number of students after 1 year of opening} = 240 \times 1.2 = 288$$

1 mark

- ii. after 3 years of opening.
- Write your answer correct to the nearest whole number.

Worked solution

$$\begin{aligned} \text{Number of students after 3 years of opening} &= 240 \times 1.2 \times 1.2 \times 1.2 \\ &= 414.72 \cong 415 \end{aligned}$$

1 mark

1 + 1 = 2 marks

- b. Write an expression that gives the number of students, K_n , the school has after n years of opening.

Worked solution

We can use the geometric sequence rule $K_n = K_0 \times r^n$, where $K_0 = 240$ and $r = 1.2$

$$K_n = 240 \times 1.2^n$$

1 mark

**Tip**

- We use $K_n = K_1 \times r^{n-1}$ rule when K_1 is given

- c. How many more students does the school have after 7 years than after 6 years of opening?

Write your answer correct to the nearest whole number.

Worked solution

$$K_7 = 240 \times 1.2^7 = 859.963\,392 \cong 860 \text{ students}$$

$$K_6 = 240 \times 1.2^6 = 716.636\,16 \cong 717 \text{ students}$$

$$K_7 - K_6 = 860 - 717 = 143 \text{ students}$$

The school will have 143 more students after 7 years than after 6 years of opening.

1 mark

- d. After how many years will the school first have more than 1000 students?

Worked solution

$$K_n = 240 \times 1.2^n \geq 1000$$

$$1.2^n \geq \frac{1000}{240}$$

$$\log 1.2^n \geq \log \left(\frac{25}{6} \right)$$

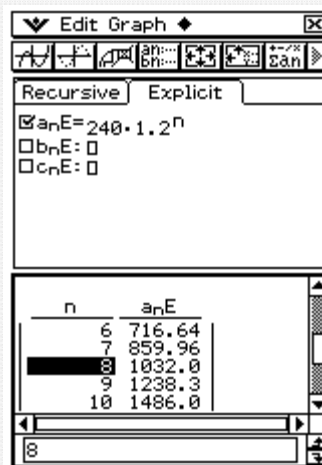
$$n \log 1.2 \geq \log \left(\frac{25}{6} \right)$$

$$n \geq \frac{\log \left(\frac{25}{6} \right)}{\log 1.2}$$

$$n \geq 7.827\,46912$$

So the school will first have more than 1000 students after 8 years.

Alternatively, we can generate the sequence by calculator.



The screenshot shows a calculator interface with a table of values for the sequence $a_n = 240 \cdot 1.2^n$. The table has two columns: n and a_n . The values for n range from 6 to 10, and the corresponding a_n values are 716.64, 859.96, 1032.0, 1238.3, and 1486.0. The row for $n=8$ is highlighted.

n	a_n
6	716.64
7	859.96
8	1032.0
9	1238.3
10	1486.0

1 mark

When Susan's school first opened it had 14 staff members. At the start of each academic year three more staff members are recruited to meet the increasing demand of the students.

A staff photo is taken at the start of each academic year. This photo is then placed on a pin board that has staff photos for all the previous years, in order by year.

- e. Susan's brother Jamie counts the total number of staff members in the first five photos to be 100. He then gets tired and asks Susan to continue counting the total number of staff members in the next five photos.

Assuming that neither Jamie nor Susan make counting errors, how many staff members does Susan count in the next five photos?

Worked solution

Let's use the arithmetic sum formula.

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} - S_5 = \frac{10}{2} [2 \times 14 + (10-1)3] - \frac{5}{2} [2 \times 14 + (5-1)3]$$

$$S_{10} - S_5 = 275 - 100 = 175$$

Alternatively, we can generate this sequence by calculator and add the values from the sixth value to the tenth value.

n	a _n E
6	29
7	32
8	35
9	38
10	41

$$\text{Sum} = 29 + 32 + 35 + 38 + 41 = 175$$

So Susan counts 175 staff members in the next five photos.

1 mark

Question 3

The yearly school tuition fee, in dollars, is generated by the difference equation

$$F_{n+1} = 0.9F_n + 2\,250 \quad F_0 = 15\,000$$

Where F_n is the yearly school tuition fee n years after the school's opening.

- a. Show that the difference equation is not geometric.

Worked solution

First of all, we need to generate the sequence by calculator.

n	a _n
0	15000
1	15750
2	16425
3	17033.
4	17579.

$$\frac{F_1}{F_0} = \frac{15\,750}{15\,000} = 1.05$$

$$\frac{F_2}{F_1} = \frac{16\,425}{15\,750} = 1.042\,857\,143$$

As $\frac{F_1}{F_0} \neq \frac{F_2}{F_1}$ this sequence is not geometric

1 mark

- b.** Use the difference equation to determine the yearly school tuition fee for a student 3 years after the school's opening.
- Write your answer correct to the nearest cent.

Worked solution

We need to determine F_3 using the difference equation.

$$F_{n+1} = 0.9F_n + 2\,250 \quad F_0 = 15\,000$$

$$\text{When } n = 0, \quad F_1 = 0.9F_0 + 2\,250 = 0.9 \times 15\,000 + 2\,250 = 15\,750$$

$$\text{When } n = 1, \quad F_2 = 0.9F_1 + 2\,250 = 0.9 \times 15\,750 + 2\,250 = 16\,425$$

$$\text{When } n = 2, \quad F_3 = 0.9F_2 + 2\,250 = 0.9 \times 16\,425 + 2\,250 = 17\,032.5$$

So the yearly school tuition fee for a student 3 years after the school's opening is \$17 032.50.

1 mark

- c.** After how many years will the yearly tuition fee of the school exceed \$20 000?

Worked solution

We had already generated the sequence by calculator in **part a**.

The screenshot shows the 'Edit Graph' window on a TI-84 Plus calculator. The 'Recursive' tab is selected. The sequence is defined as $a_{n+1} = 0.9 \cdot a_n + 2250$ with $a_0 = 15000$. Below the definition, there are empty boxes for b_{n+1} , c_{n+1} , and d_{n+1} . At the bottom, a table of values is displayed with columns n and a_n . The values for n are 8, 9, 10, 11, and 12. The corresponding values for a_n are 19271., 19594., 19885., 20146., and 20382. The row for $n=11$ is highlighted.

n	a_n
8	19271.
9	19594.
10	19885.
11	20146.
12	20382.

The yearly tuition fee of the school will exceed \$20 000 after 11 years.

1 mark

- d. What is the total amount of school tuition fees that the school collected in the first 2 years?

Worked solution

In year 1:

Initially, there were 240 students at the school and each student paid \$15 000 for school tuition fees.

$$\begin{aligned}\text{Total amount of school tuition fees collected in the first year} &= 240 \times \$15\,000 \\ &= \$3\,600\,000.\end{aligned}$$

In year 2:

There were 288 students at the school after 1 year of opening and each student paid \$15 750 for school tuition fees.

$$\begin{aligned}\text{Total amount of school tuition fees collected in the second year} &= 288 \times \$15\,750 \\ &= \$4\,536\,000\end{aligned}$$

$$\text{Total} = \$3\,600\,000 + \$4\,536\,000 = \$8\,136\,000$$

So the total amount of school tuition fees that the school collected in the first 2 years is \$8 136 000.

2 marks

Mark allocation

- 1 mark for calculating the total amount of school tuition fees collected in the first year.
- 1 mark for the correct answer.

Total 15 marks

**END OF MODULE 1
TURN OVER**

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Module 2: Geometry and trigonometry

Question 1

Juliet is an interior designer and her new project is painting the interior walls of an arts centre.

The centre has a triangular shape, as shown in Figure 1 below.

The wall AB runs in an east–west line and is 56 metres long.

The wall BC is 45 metres long.

The angle between these two walls is 42° .

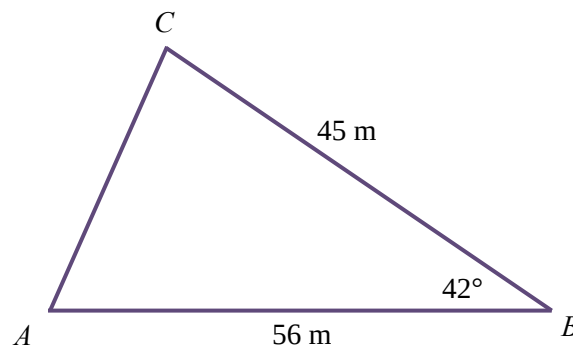


Figure 1

- a. Find the length of the wall AC .
Write your answer in metres, correct to two decimal places.

Worked solution

Let's apply the cosine rule to the triangle to find the length of the wall $AC = x$.

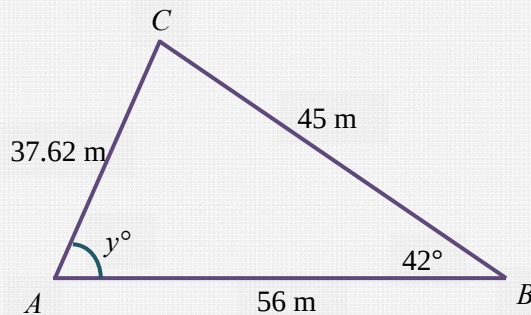
$$x^2 = 45^2 + 56^2 - 2 \times 45 \times 56 \times \cos 42^\circ$$

$$x = 37.623\,796\,72 \text{ m} \cong 37.62 \text{ m}$$

1 mark

- b. Using your answer from question 1a, find the magnitude of the angle BAC .
Express your answer correct to the nearest degree.

Worked solution



Let's apply the sine rule to the triangle to find the magnitude of the angle $BAC = y$.

$$\frac{45}{\sin y^\circ} = \frac{37.62}{\sin 42^\circ}$$

$$\sin y = \frac{45 \times \sin 42^\circ}{37.62}$$

$$\sin y = 0.800\,395\,462\,151\,744$$

$$y = \sin^{-1} 0.800\,395\,462\,151\,744 = 53.167\,882\,8^\circ \approx 53^\circ$$

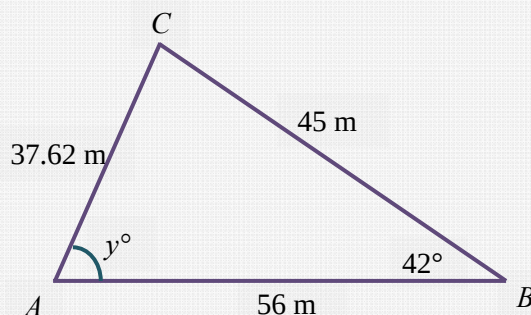
2 marks

Mark allocation

- 1 mark for the correct application of the sine rule.
- 1 mark for the correct answer.

- c. Find the area of the arts centre, correct to the nearest square metre.

Worked solution



Let's calculate the area of the triangle using the rule:

$$\text{Area} = \frac{1}{2} \times a \times c \times \sin A$$

$$= \frac{1}{2} \times 45 \times 56 \times \sin 42^\circ$$

$$= 843.145\,64 \text{ m}^2 = 843 \text{ m}^2$$

1 mark

Question 2

When she had finishing painting the walls, Juliet divided the arts centre into two sections, as shown in Figure 2 below.

She then started decorating section 2 with Aboriginal motives.

The length of the wall DB is 33.44 metres and the new wall CD runs in a north–south line.

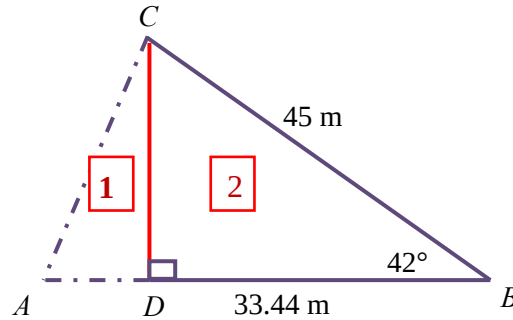
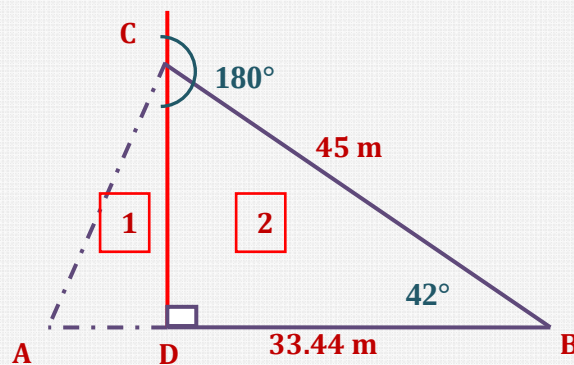


Figure 2

- a. Find the bearing of D from C .

Worked solution



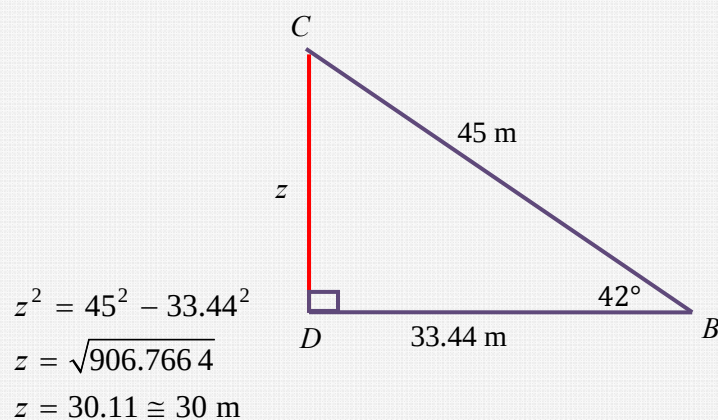
The bearing of D from C is 180°T .

1 mark

- b. Find the length of the new wall CD , correct to the nearest metre.

Worked solution

We will first use Pythagoras' theorem to find the length of the side $CD = z$.



Alternatively we can use the trigonometric ratios to determine the length of the side CD .

$$\sin 42^\circ = \frac{z}{45}$$

$$z = 45 \times \sin 42^\circ$$

$$z = 30.11 \approx 30 \text{ m}$$

1 mark

- c. Juliet made a scale diagram of section 2 to draw better plans for decorating.
The ratio of the area of section 2 on Juliet's diagram to the actual area is 1:121.
What was the length of the wall DB on Juliet's diagram?
Write your answer in centimetres.

Worked solution

$$\text{Area scale factor} = k^2 = \frac{1}{121}$$

$$\text{Length scale factor} = k = \sqrt{\frac{1}{121}} = \frac{1}{11}$$

$$\text{Length scale factor} = k = \frac{\text{length of the wall } DB \text{ on the diagram}}{\text{actual length of the wall } DB}$$

$$= \frac{\text{length of the wall } DB \text{ on the diagram}}{33.44 \text{ m}} = \frac{1}{11}$$

$$\text{length of the wall } DB \text{ on the diagram} = \frac{33.44 \text{ m} \times 1}{11}$$

$$\text{the length of the wall } DB \text{ on the diagram} = 3.04 \text{ m} = 304 \text{ cm}$$

2 marks

Mark allocation

- 1 mark for calculating the value of length scale factor, k .
- 1 mark for the correct answer

Question 3

The arts centre is located at an altitude of 15 metres on a hillside. A contour map of the hill is shown in Figure 3 below. During a lunch break while working on her project, Juliet goes to the bank, which is located at an altitude of 45 metres, to withdraw money. She then goes to the post office, which is located at an altitude of 0 metres, to pay her bills.

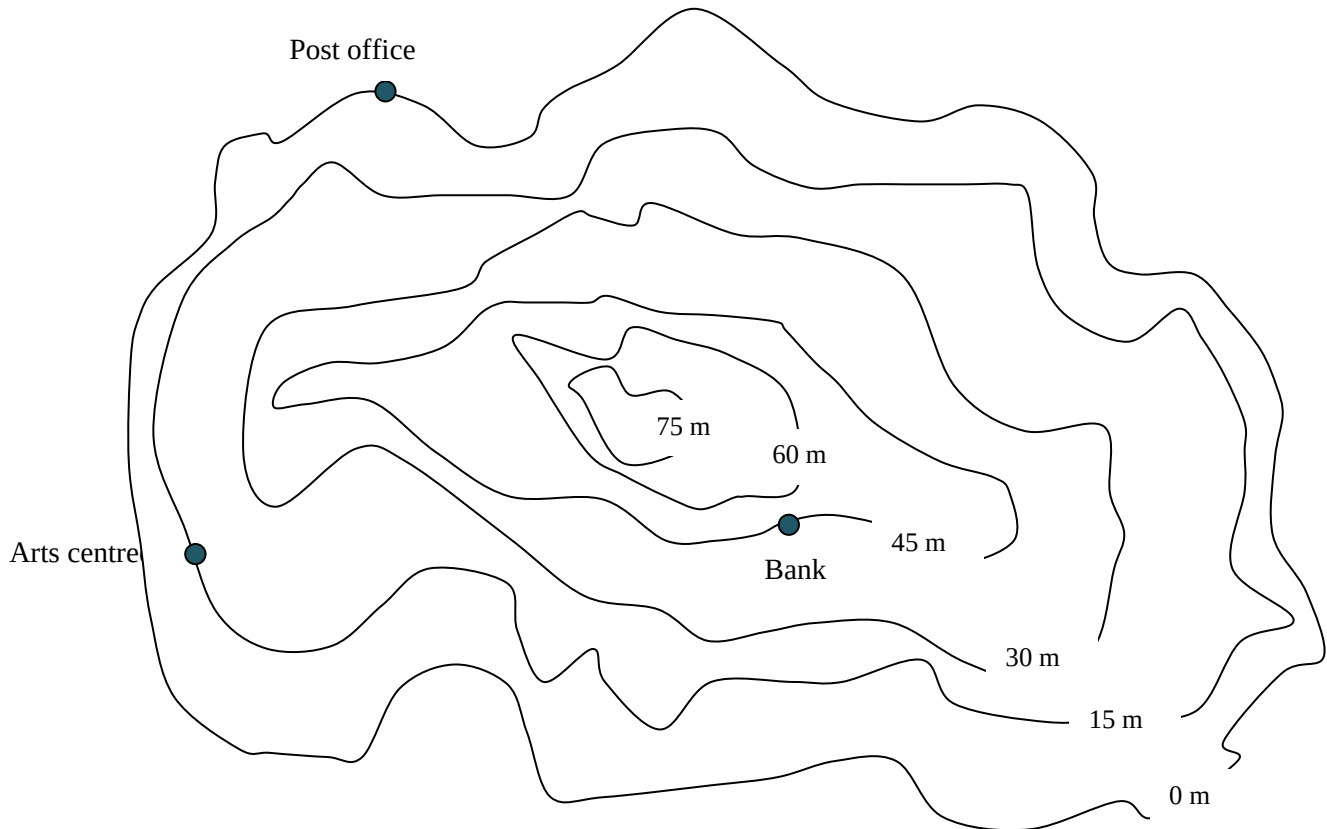


Figure 3

On the contour map, 1 cm represents 10 metres on the horizontal level.

- a.** Determine the vertical distance between the arts centre and the bank.

Worked solution

The bank is located at an altitude of 45 metres and the arts centre is located at an altitude of 15 metres.

The vertical distance between the arts centre and the bank is: $45 \text{ m} - 15 \text{ m} = 30 \text{ m}$

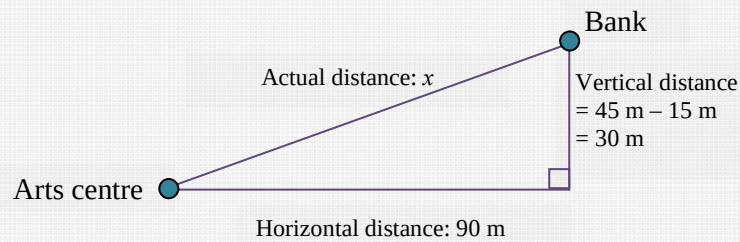
1 mark

- b. The map distance between the arts centre and the bank is given as 9 centimetres. Determine the actual distance, in metres, from the arts centre to the bank. Write your answer in metres, correct to three decimal places.

Worked solution

1 cm represents 10 m on the horizontal level, so 9 cm represents $9 \times 10 = 90$ m

The horizontal distance between the arts centre and the bank is 90 m.



Now let's use Pythagoras' theorem to determine the value of x .

$$x^2 = 30^2 + 90^2$$

$$x^2 = 9\,000$$

$$x = \sqrt{9\,000}$$

$$x = 94.868\,329\,81 \approx 94.868 \text{ m}$$

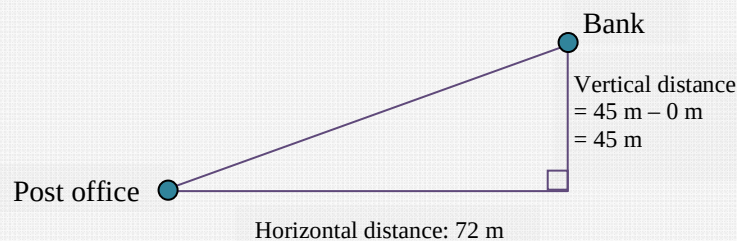
2 marks

Mark allocation

- 1 mark for calculating the horizontal distance between the arts centre and the bank.
- 1 mark for the correct answer

- c. The horizontal distance between the bank and the post office is 72 metres. Calculate the average slope between the bank and the post office.

Worked solution



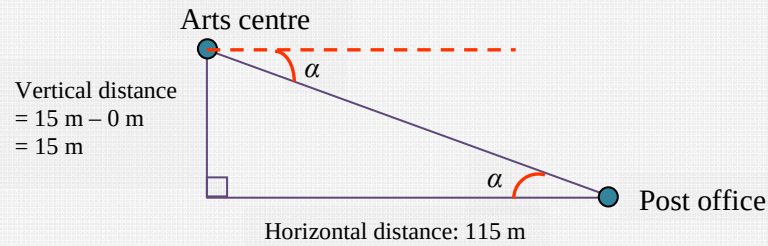
$$\text{average slope} = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{45 \text{ m}}{72 \text{ m}} = 0.625$$

1 mark

- d. Map distance between the post office and the arts centre is given as 11.5 centimetres. Calculate the angle of depression of a person looking at the post office from the arts centre.
Write your answer in degrees, correct to two decimal places.

Worked solution

1 cm represents 10 m on the horizontal level, so 11.5 cm represents $11.5 \times 10 = 115$ m.
So, horizontal distance between the post office and the arts centre is 115 m.



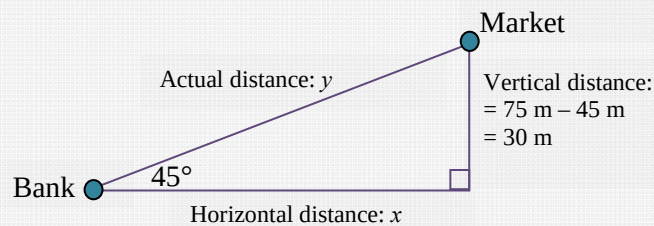
Angle of depression = α

$$\tan \alpha = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{15 \text{ m}}{115 \text{ m}}$$

$$\alpha = \tan^{-1} \frac{15}{115} = 7.4314^\circ \cong 7.43^\circ$$

1 mark

- e. On another day Juliet goes to the bank and then to the market.
 The market is located at an altitude of 75 metres.
 The angle of elevation of the market from the bank is 45° .
 Determine the horizontal distance and the actual distance between the bank and the market.
 Write your answers in metres, correct to one decimal place.

Worked solution

$$\tan 45^\circ = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

$$\tan 45^\circ = \frac{30 \text{ m}}{\text{horizontal distance}}$$

$$\text{horizontal distance} = \frac{30}{\tan 45^\circ}$$

$$\text{horizontal distance} = 30.0 \text{ m}$$

We can use Pythagoras' theorem to calculate the actual distance between the bank and the market.

$$y^2 = 30^2 + 30^2$$

$$y^2 = 1800$$

$$y = \sqrt{1800}$$

$$y = 42.4264 \text{ m} \approx 42.4 \text{ m}$$

Alternatively, we can use the trigonometric ratios to determine the actual distance between the bank and the market.

$$\sin 45^\circ = \frac{30}{y}$$

$$y = \frac{30}{\sin 45^\circ}$$

$$y = 42.4 \text{ m}$$

2 marks

Mark allocation

- 1 mark for calculating the horizontal distance between the bank and the market.
- 1 mark for the correct answer

Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

The towns of Lorne and Apollo Bay are 45 km apart. A cyclist, Margaret, leaves Lorne and travels towards Apollo Bay.

- a. Margaret cycles at a constant speed of 9 km/h for the first 3 hours without taking any rest periods. How far does she travel in the first 3 hours of her journey?

Worked solution

distance = speed $\times$ time

$$\text{distance} = 9 \times 3 = 27 \text{ km}$$

Margaret travels 27 km in the first 3 hours of her journey.

1 mark

- b. After cycling for 3 hours, Margaret decides to stop for a 30 minute lunch break. She then cycles at a constant speed of 12 km/h for the rest of her journey.
How long does her journey from Lorne to Apollo Bay take?

Worked solution

Let's calculate the time for the last leg of her journey.

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{45 \text{ km} - 27 \text{ km}}{12 \text{ km/h}} = \frac{18}{12} = 1.5 \text{ hours}$$

So total time for the journey is $3 + \frac{30}{60} + 1.5 = 5$ hours.

1 mark

- c. What is Margaret's average speed, in kilometres per hour, during her journey from Lorne to Apollo Bay?

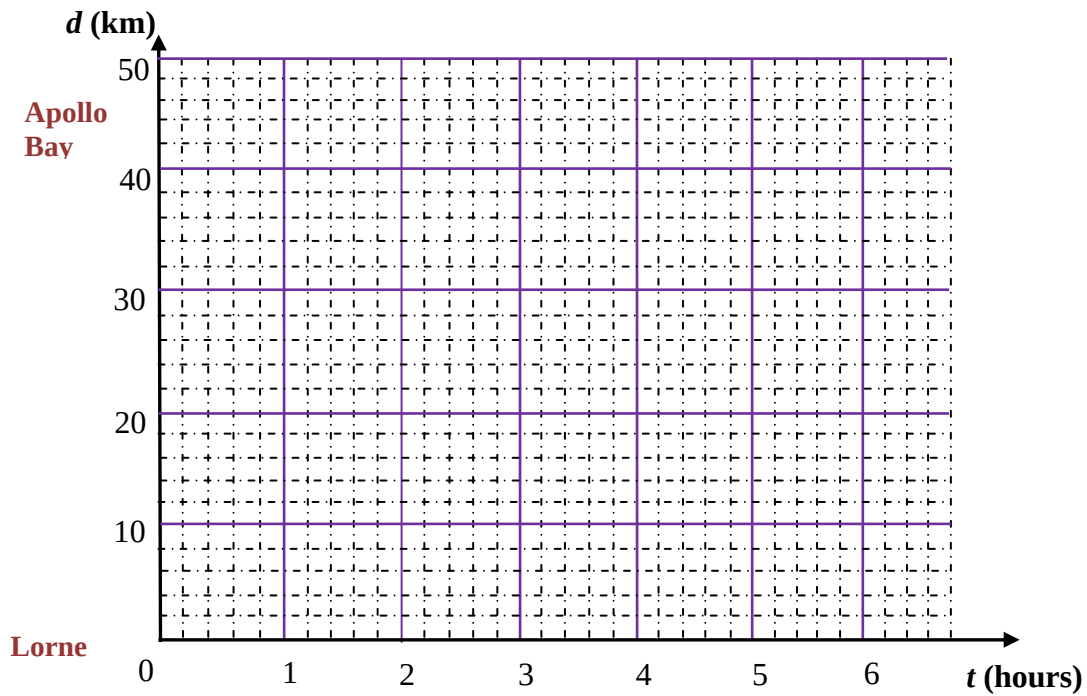
Worked solution

$$\text{average speed} = \frac{\text{total distance covered}}{\text{total time taken}} = \frac{45}{5} = 9 \text{ km/h}$$

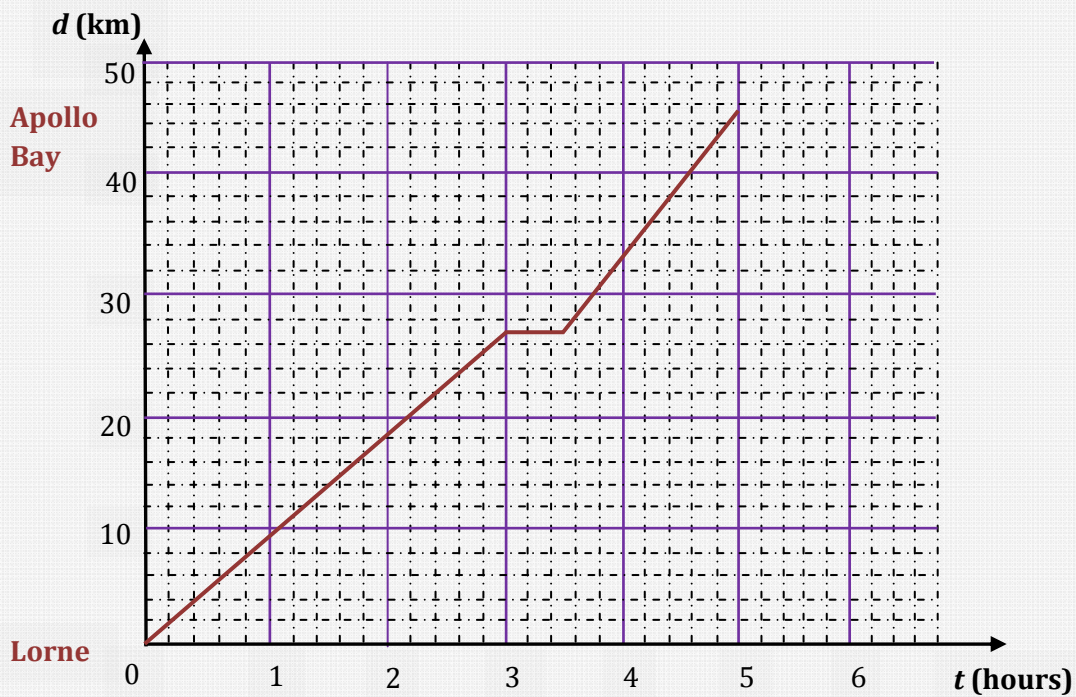
1 mark

- d. Let d represent the distance cycled by Margaret and t the time taken during her journey.

Draw and label the graph of d (km) against t (hours) on the set of axes below.



Worked solution



1 mark

Another cyclist, Richard, leaves Apollo Bay and travels towards Lorne. Richard and Margaret start their journeys towards each other at the same time.

The remaining distance of Richard from Lorne, in kilometres, is given by the equation $d = 45 - 7.5t$ for $0 \leq t < k$, where t is the time, in hours, since he began cycling.

- e. Determine the value of k .

Worked solution

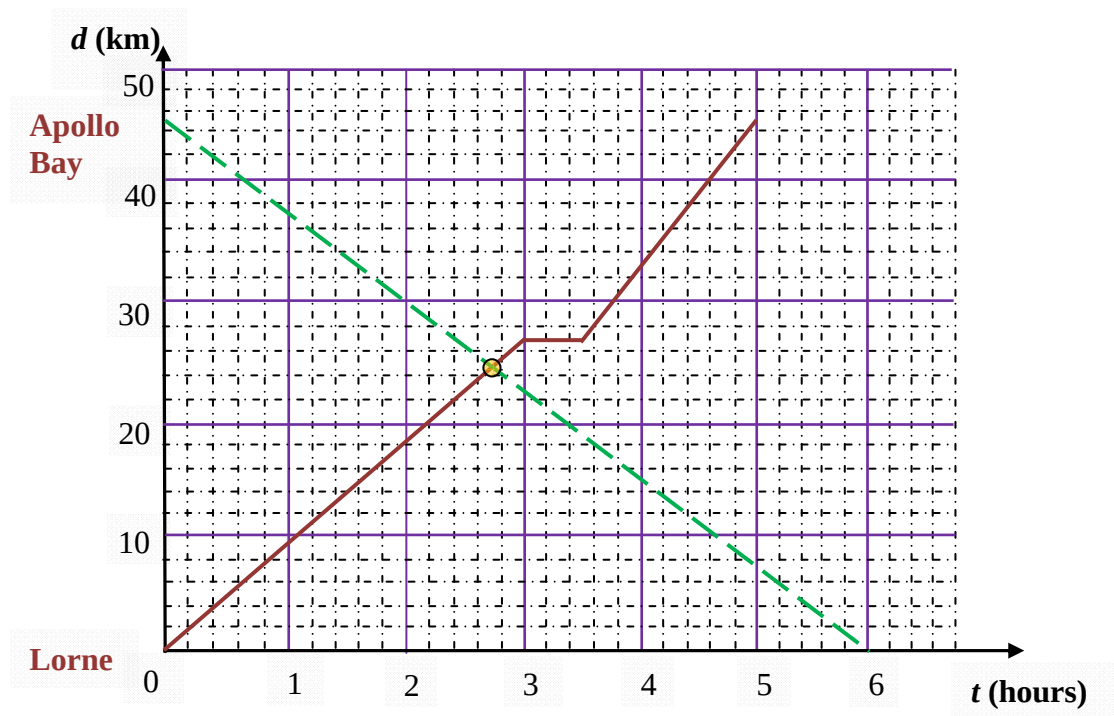
Worked solution

When $d = 0$

$$0 = 45 - 7.5 \times k$$

So, $k = 6$ hours

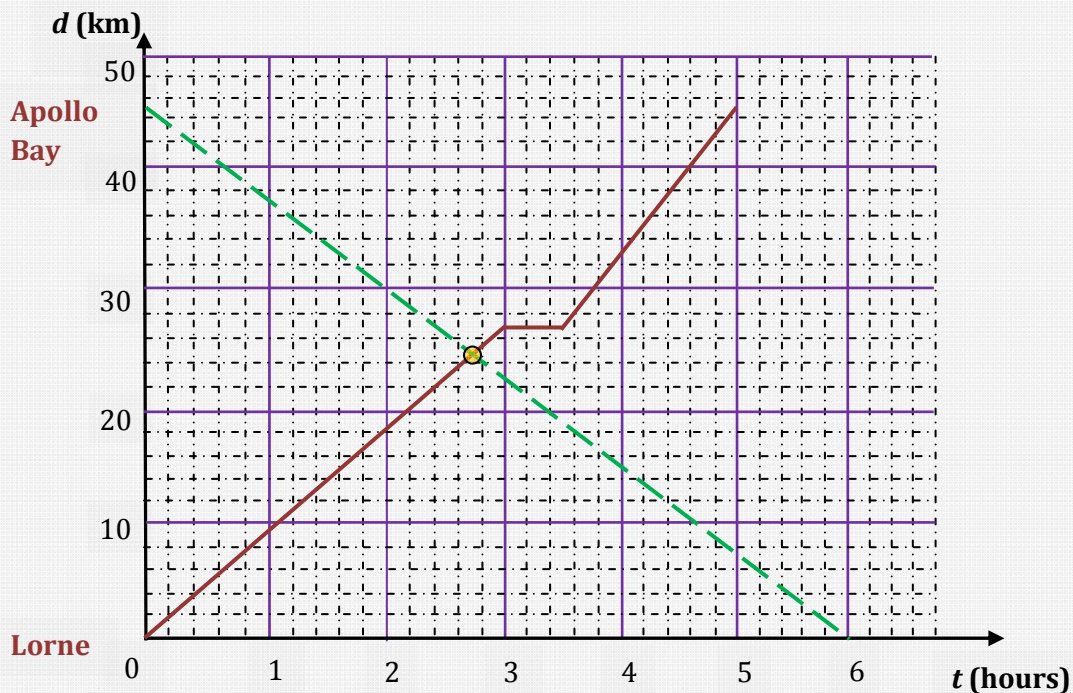
1 mark



- f. After how many hours did their paths cross?
Write your answer correct to one decimal place.

Worked solution

Let's first draw the second graph on the same set of axes as the first graph to see the point of intersection.



Now we can solve two equations simultaneously to determine the coordinates of the intersection point.

$$d = 9t$$

$$d = 45 - 7.5t$$

$$t = 2.727\ 272\ 72 \approx 2.7 \text{ hours}$$

1 mark

- g. Where did they pass each other?
Write your answer correct to one decimal place.

Worked solution

Let's substitute $t = 2.727\ 272\ 72 \approx 2.7$ hours into the equation $d = 9t$

$$d = 9 \times 2.727\ 272\ 72 = 24.545\ 454\ 545\ 45 \approx 24.5 \text{ km}$$

So they pass each other at 24.5 km from Lorne, or at 20.5 km from Apollo Bay.

1 mark

Question 2

A skip bin hire company has two different types of skip bins for hire: small bins and large bins.

Let x be the number of small skip bins that are hired each day.

Let y be the number of large skip bins that are hired each day.

A constraint on the number of skip bins that can be hired each day is given by

Inequality 1: $x + y \leq 7$

- a. Explain the meaning of Inequality 1 in terms of the context of this problem.

Worked solution

The total number of small and large skip bins that are hired each day must not exceed 7.

1 mark

Each small bin requires four people to deliver it and each large bin requires five people.

There are only five small bins and three large bins available to hire each day.

There are only 24 people available to do the delivery of the skip bins each day.

These constraints are given by

Inequality 2: $4x + 5y \leq 24$

Inequality 3: $x \geq 0$

Inequality 4: $y \geq 0$

Inequality 5: $x \leq 5$

Inequality 6:

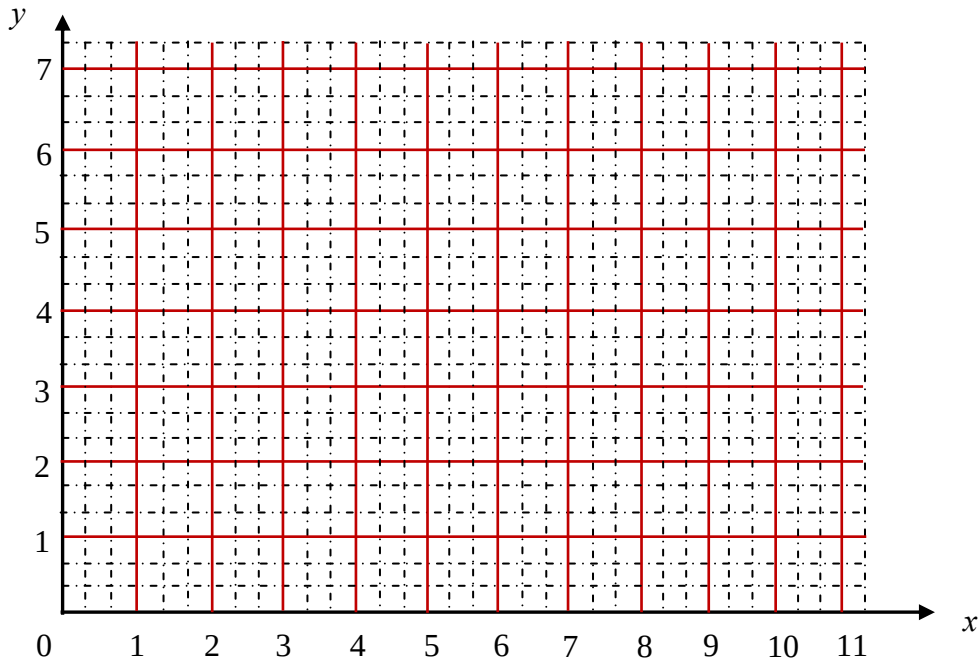
- b. Fill in the sixth constraint on the number of the large skip bins that can be hired each day.

Worked solution

Inequality 6: $y \leq 3$

1 mark

- c. Draw the line that represents $4x + 5y = 24$ on the graph below.



Worked solution

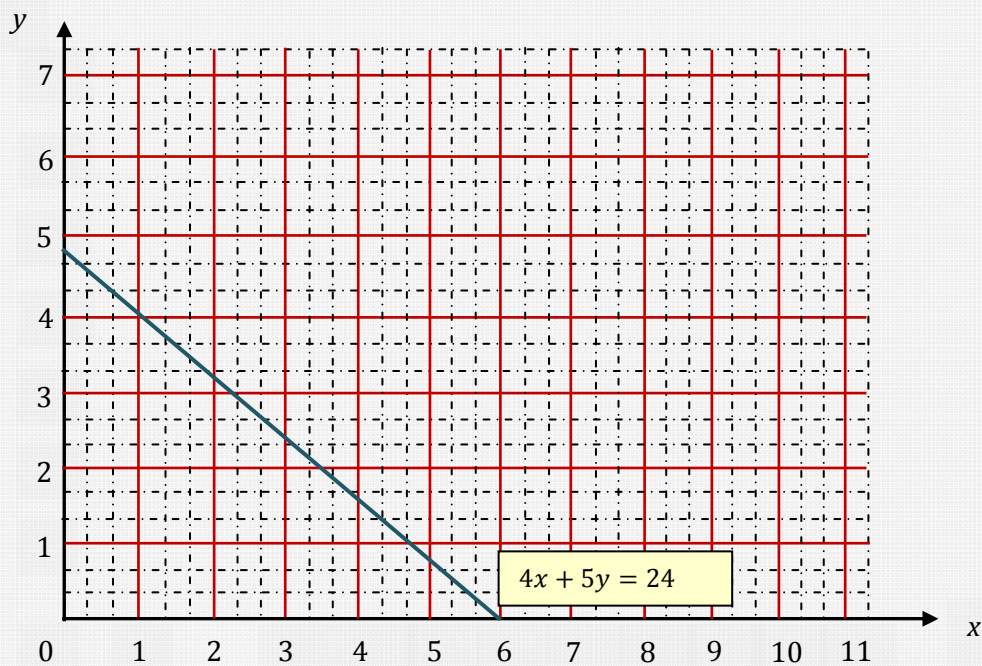
Let's evaluate the x - and y -intercepts.

$$x = 0 \quad 4 \times 0 + 5y = 24$$

$$\therefore y \text{ intercept} = 4.8$$

$$y = 0 \quad 4x + 5 \times 0 = 24$$

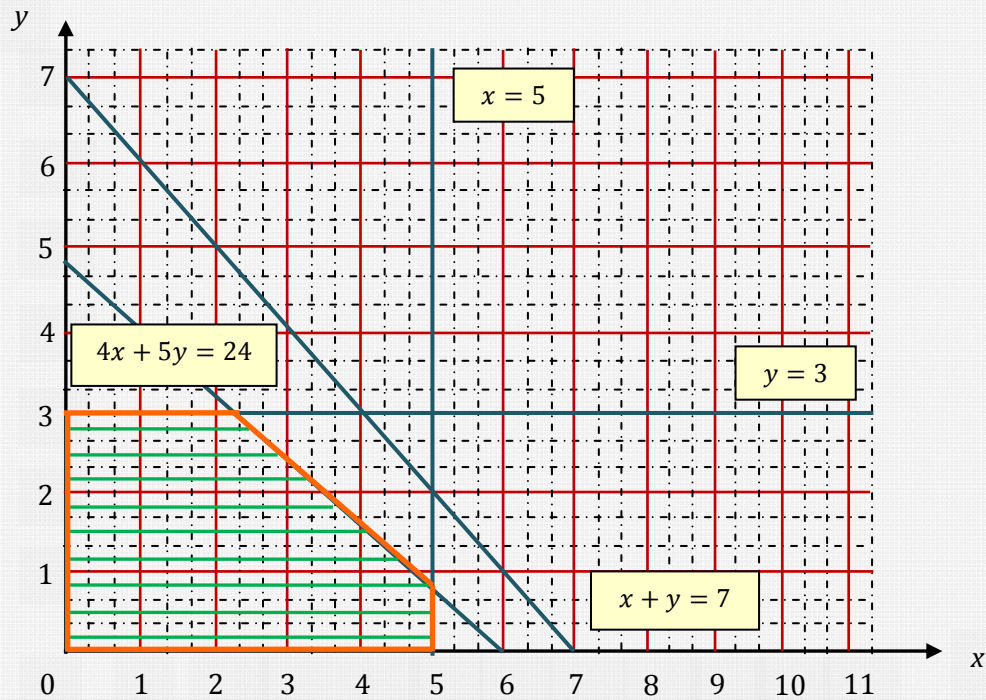
$$\therefore x \text{ intercept} = 6$$



1 mark

- d. i. On the graph given in **part c**, draw and clearly indicate the boundaries of the region represented by inequalities 1 to 6. Shade the feasible region.

Worked solution



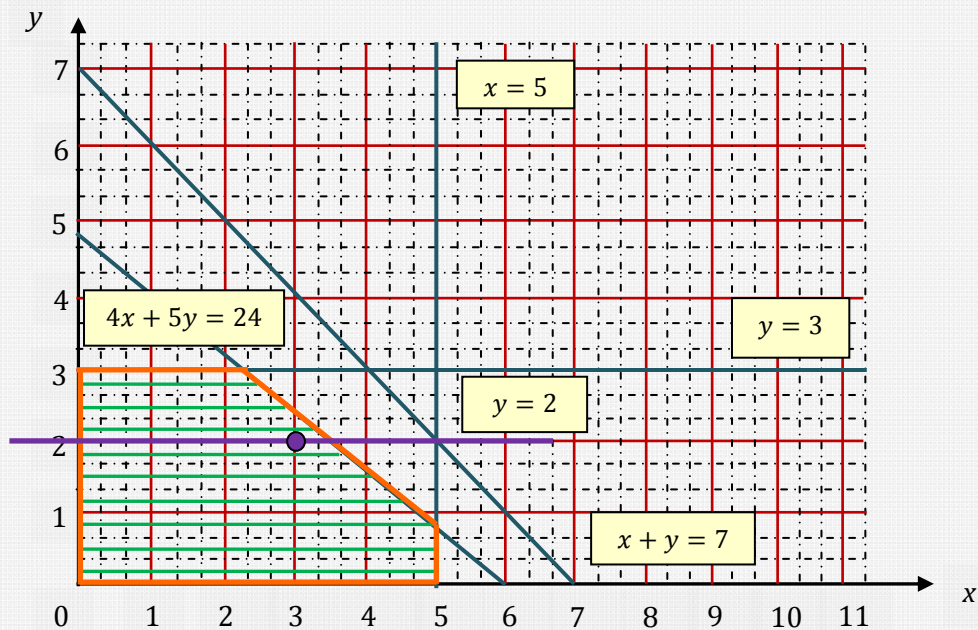
2 marks

Mark allocation

- 1 mark for drawing all the required lines correctly.
- 1 mark for shading the feasible region.

- ii. On a day when exactly two large skip bins are hired, what is the maximum number of small bins that could be hired?

Worked solution



We can clearly observe from the feasible region of the above graph that when y value is 2, the maximum possible x value is 3.

1 mark

Mark allocation

- 1 mark for drawing all the required lines correctly.

2 + 1 = 3 marks

The profit from a small skip bin hire is \$70 and the profit from a large skip bin hire is \$110. Let P be the total profit obtained from small and large skip bin hires.

- e. Write an equation for the total profit, P , in terms of x and y .

Worked solution

$$P = 70x + 110y$$

1 mark

- f. Determine the maximum total profit that the skip bin hire company can make per day from the hire of these skip bins.

Worked solution

The number of skip bins that can be hired in a day is a discrete variable, so we need to look for integer values; that is, whole numbers representing the number of small skip bins and large skip bins.

In the feasible region of the graph, which is shaded, there are 18 points that satisfy the constraints.

The feasible points are

(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (5, 0)

Let's substitute the points that would yield the maximum possible profit in the profit equation.

$$P = 70x + 110y$$

$$(2, 3) \rightarrow P = 70 \times 2 + 110 \times 3 = 470 \text{ (maximum profit)}$$

$$(3, 2) \rightarrow P = 70 \times 3 + 110 \times 2 = 430$$

$$(4, 1) \rightarrow P = 70 \times 4 + 110 \times 1 = 390$$

$$(5, 0) \rightarrow P = 70 \times 5 + 110 \times 0 = 350$$

So, the maximum total profit that the skip bin hire company can make per day from these skip bins is \$470.

1 mark

Total 15 marks

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Module 4: Business-related mathematics

Question 1

Jessica invests \$100 000 into a bank account that earns a compound interest of 5.2%, compounding annually. She does not make any deposits or withdrawals for 20 years.

- a. Determine the total value of this investment at the end of 20 years.
Write your answer correct to the nearest dollar.

Worked solution

$$A = P \times R^n \text{ and } R = 1 + \frac{r}{100} \text{ where } r = 5.2$$

$$A = 100\,000 \times \left(1 + \frac{5.2}{100}\right)^{20} = \$275\,622.5817 \approx \$275\,623$$

Alternatively, we can enter the given values into the calculator and solve it by TVM solver.

The screenshot shows a calculator interface with a 'Compound Interest' section. The fields are filled with the following values:

Field	Value
N	20
I%	5.2
PV	-100000
PMT	0
FV	275622.5817
P/Y	1
C/Y	1

At the bottom of the screen, there are buttons for 'Help' and 'Format'.

1 mark

- b. If Jessica had invested the \$100 000 in an account that earned 5.2% per annum simple interest, what would the investment have been worth after 20 years?

Worked solution

Now let's calculate the simple interest earned by using the simple interest formula.

$$I = \frac{Prt}{100}$$

$$I = \frac{100\,000 \times 5.2 \times 20}{100} = \$104\,000$$

After 20 years, Jessica's investment would have been worth

$$\$100\,000 + \$104\,000 = \$204\,000$$

1 mark

- c. If Jessica had invested the \$100 000 in an account that earned 5.2% per annum compound interest, compounding monthly, what would the investment have been worth after 20 years?

Write your answer correct to the nearest cent.

Worked solution

$$A = P \times R^n \text{ and } R = 1 + \frac{r}{100} \text{ where } r = 5.2$$

$$A = 100\,000 \times \left(1 + \frac{5.2/12}{100}\right)^{240} = \$282\,286.7339 \approx \$282\,286.73$$

Alternatively, we can enter the given values into the calculator and solve it by TVM solver.

The screenshot shows a calculator interface with a 'Compound Interest' section. The fields are filled with the following values:

Field	Value
N	240
I%	5.2
PV	-100000
PMT	0
FV	282286.7339
P/Y	12
C/Y	12

At the bottom, there are buttons for 'Help' and 'Format'.

1 mark

- d. Jessica's friend Monica has also invested \$100 000 into an account that earns a compound interest of 5.2% per annum, compounding quarterly, and makes no further deposits or withdrawals to it.

If Monica invested her money into this account at the start of January 2012, when would the amount in this account first be more than double?

Worked solution

$$A = P \times R^n \text{ and } R = 1 + \frac{r}{100} \text{ where } r = 5.2$$

$$A = 100\,000 \times \left(1 + \frac{5.2/4}{100}\right)^n = \$200\,000$$

$$100\,000 \times 1.013^n = 200\,000$$

$$1.013^n = 2$$

$$\log 1.013^n = \log 2$$

$$n \log 1.013 = \log 2$$

$$n = \frac{\log 2}{\log 1.013} = 53.664\,841\,41 \cong 54 \text{ months}$$

After 53.66 months $\cong$ 54 months = 4 years and 6 months the account first doubles.

So, the amount in this account would be more than double at the start of July 2016.

Alternatively, we can enter the given values into the calculator and solve it by TVM solver.

Compound Interest	
N	53.66484141
I%	5.2
PV	-100000
PMT	0
FV	200000
P/Y	4
C/Y	4

2 marks

Mark allocation

- 1 mark for finding 54 months with the correct working out (or using the TVM solver values).
- 1 mark for the correct answer

Question 2

Jessica's brother, Ben, takes out a reducing balance loan of \$750 000 at an interest rate of 7.4% per annum, compounding monthly, to buy a new house.

- a. Ben is planning to repay his loan in full in 10 years.
- i. What is the monthly repayment he needs to pay?
Write your answer correct to the nearest cent.

Worked solution

We use TVM solver to find Ben's monthly repayments.

The screenshot shows a 'Compound Interest' TVM solver window. The fields are filled with the following values:

Field	Value
N	120
I%	7.4
PV	750000
PMT	-8863.537504
FV	0
P/Y	12
C/Y	12

At the bottom, there are buttons for 'Help', 'Format', 'Solve', and 'End'.

Ben will have to pay \$8 863.54 a month to be able to repay his loan in full in 10 years.

1 mark

- ii. Calculate the total interest that Ben will pay over the 10-year period.
Write your answer correct to the nearest dollar.

Worked solution

Ben will be repaying $8\,863.54 \times 120 = \$1\,063\,624.8 \approx \$1\,063\,625$ over the 10 year period .

The total interest = $\$1\,063\,625 - \$750\,000 = \$313\,625$

So he will pay \$313 625 as interest over 10 years.

1 mark

1 + 1 = 2 marks

- b. Suppose, instead, Ben pays \$6 000 each month for the first 6 years of the loan and then he increases his payments to \$7 000 per month for the following 4 years. Find the amount outstanding on the loan at the end of the 10-year period. Write your answer correct to the nearest cent.

Worked solution

Let's calculate the amount outstanding after 6 years of monthly payments of \$6 000 by TVM solver.

The TVM Solver screen displays the following values:

Variable	Value
N	72
I%	7.4
PV	750000
PMT	-6000
FV	-625847.2012
P/Y	12
C/Y	12

Buttons at the bottom: Help, Format, Solve, End.

So, amount outstanding after 6 years is \$625 847.201 2.

Now let's calculate the amount outstanding after 4 years of monthly payments of \$7 000 by TVM solver.

The TVM Solver screen displays the following values:

Variable	Value
N	48
I%	7.4
PV	625847.2012
PMT	-7000
FV	-451034.79
P/Y	12
C/Y	12

Buttons at the bottom: Help, Format, Solve, End.

So, the amount outstanding on the loan at the end of the 10-year period is \$451 034.79.

2 marks

Mark allocation

- 1 mark for calculating the amount outstanding after 6 years.
- 1 mark for the correct answer

Question 3

Ben purchases a car with a discounted price of \$35 000.

- a. The original price of the car before discount was \$40 000. What is the percentage discount that Ben received?

Worked solution

$$\text{percentage discount} = \frac{\text{original price} - \text{new price}}{\text{original price}} \times 100\%$$

$$\text{percentage discount} = \frac{40\,000 - 35\,000}{40\,000} \times 100\%$$

$$\text{percentage discount} = 12.5\%$$

1 mark

- b. Ben agrees to a hire-purchase contract for the car costing \$35 000, which requires a deposit of \$15 000 and 60 equal monthly payments of \$550.

Find the effective rate of interest per annum that he is paying for this contract.

Express your answer as a percentage, correct to two decimal places.

Worked solution

$$\text{total paid} = \$15\,000 + 60 \times \$550 = \$48\,000$$

$$\text{interest paid} = \$48\,000 - \$35\,000 = \$13\,000$$

$$r_e = \frac{100I}{Pt} \times \frac{2n}{n+1}$$

$$r_e = \frac{100 \times 13\,000}{(35\,000 - 15\,000) \times 5} \times \frac{2 \times 60}{60 + 1}$$

$$r_e = 25.573\,77\% \cong 25.57\%$$

1 mark

- c. The car that Ben purchases for \$35 000 is to be depreciated over time for taxation purposes. If the car is depreciated by 20 cents per kilometre, how many kilometres will the car travel before its book value is reduced to \$20 000?

Worked solution

Book value = original value – unit cost depreciation amount $\times$ no. of kilometres

$$\$20\,000 = \$35\,000 - \$0.2x$$

$$0.2x = 35\,000 - 20\,000$$

$$0.2x = 15\,000$$

$$x = 75\,000 \text{ km}$$

1 mark

- d. If the car is depreciated by 5% of its purchase price each year, what is the book value of the car after 3 years?

Worked solution

$$5\% \text{ of } \$35\,000 \text{ is } \$35\,000 \times \frac{5}{100} = \$1\,750$$

$$\text{The value of the car after 3 years} = \$35\,000 - 3 \times \$1\,750 = \$29\,750$$

1 mark

- e. If the value of the car depreciates at a rate of 6% per annum, by how much will the car depreciate during the fifth year?

Write your answer correct to the nearest dollar.

Worked solution

$$V = P \times \left(1 - \frac{r}{100}\right)^t$$

$$\text{When } t = 4, \quad V = 35\,000 \times \left(1 - \frac{6}{100}\right)^4 = \$27\,326.213\,6$$

$$\text{When } t = 5, \quad V = 35\,000 \times \left(1 - \frac{6}{100}\right)^5 = \$25\,686.640\,78$$

$$\begin{aligned} \text{Depreciation amount during the fifth year} &= \$27\,326.213\,6 - \$25\,686.640\,78 \\ &= \$1\,639.572\,82 \cong \$1\,640 \end{aligned}$$

2 marks

Mark allocation

- You can allocate 1 mark for the correct application of the reducing balance depreciation formula.
- 1 mark for the correct answer

Total 15 marks

Module 5: Networks and decision mathematics

Question 1

Josh, an information technology technician, needs, to visit nine houses on Monday. The network below shows the roads that are used to travel between the houses, which are labelled from M to V (see Figure 1).

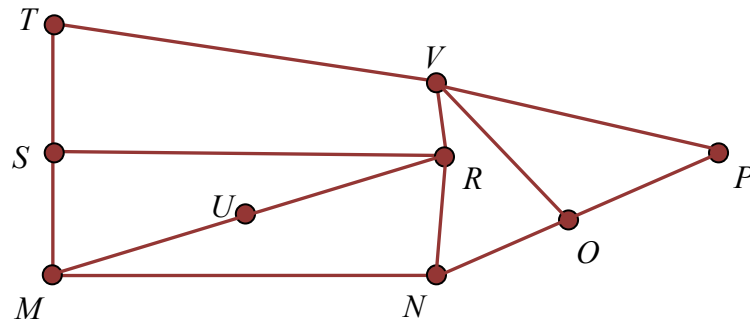


Figure 1

- a. Find the sum of the degrees of the vertices of the network shown in Figure 1.

Worked solution

Sum of the degrees of the vertices = $3 + 3 + 2 + 4 + 4 + 2 + 3 + 3 + 2 = 26$

1 mark

- b. Write down a Hamiltonian circuit that starts and finishes at U .

Worked solution

A **Hamiltonian path** is a path through a graph that passes through each vertex exactly once.

A **Hamiltonian circuit** is a Hamiltonian path that starts and finishes at the same vertex.

Some examples of Hamiltonian circuits:

First Hamiltonian circuit: $U-M-S-T-V-P-O-N-R-U$

Second Hamiltonian circuit: $U-M-N-O-P-V-T-S-R-U$

Third Hamiltonian circuit: $U-R-N-O-P-V-T-S-M-U$

1 mark

- c. Verify that the graph has six faces, using Euler's formula.

Worked solution

The graph has 9 vertices and 13 edges.

Euler's formula states:

$$v - e + f = 2 \quad \text{where } v = 9 \text{ and } e = 13$$

$$f = 2 - v + e$$

$$f = 2 - 9 + 13$$

$$f = 6$$

1 mark

- d. Josh wants to start at point M and travel each road exactly once, finishing at point M . He realises that this is impossible. Explain why such a circuit does not exist.

Worked solution

A path that includes every edge just once is called an **Euler path**.

An **Euler circuit** is an Euler path that starts and finishes at the same vertex.

An Euler circuit exists when all vertices of the connected graph have an even degree.

An Euler circuit does not exist for this graph because not all vertices are of even degree.

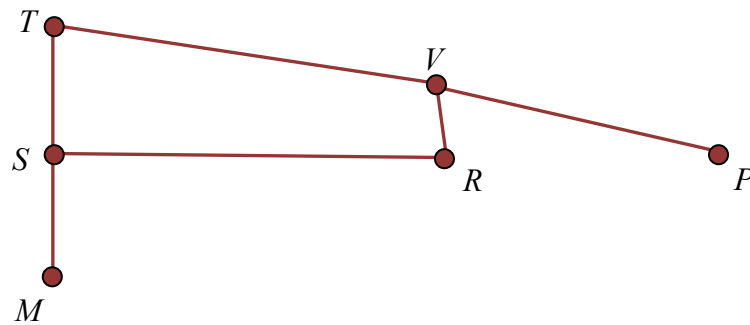
2 marks

Mark allocation

- 1 mark for the correct explanation of an Euler circuit.
- 1 mark for giving the reason why an Euler circuit does not exist.

Question 2

On Tuesday, Josh gets sick and sends his partner Gary to complete the jobs with the highest priority. These houses and the roads linking them are shown below in Figure 2.

**Figure 2**

The adjacency matrix below indicates the houses that have a road running directly between them.

$$\begin{array}{c}
 \begin{array}{c} M \\ P \\ R \\ S \\ T \\ V \end{array}
 \begin{array}{c}
 M \quad P \quad R \quad S \quad T \quad V \\
 \left[\begin{array}{cccccc}
 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 \\
 a & 0 & 0 & 1 & 0 & 1 \\
 1 & 0 & b & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 & 0 & c \\
 0 & 1 & 1 & 0 & 1 & 0
 \end{array} \right]
 \end{array}
 \end{array}$$

- a. Explain why $c = 1$.

Worked solution

$c = 1$ because there is one road between T and V .

1 mark

- b.** Find the sum of the values a and b in the adjacency matrix above.

Worked solution

There is no road between R and M , so $a = 0$

There is one road between S and R , so $b = 1$

So, $a + b = 0 + 1 = 1$

1 mark

- c.** What is the sum of the elements in column 4 of the adjacency matrix and what does this tell us in the context of this problem?

Worked solution

Sum of the elements in column 4 = $1 + 0 + 1 + 0 + 1 + 0 = 3$

This means that house S is linked to three houses, T , R and M by roads.

1 mark

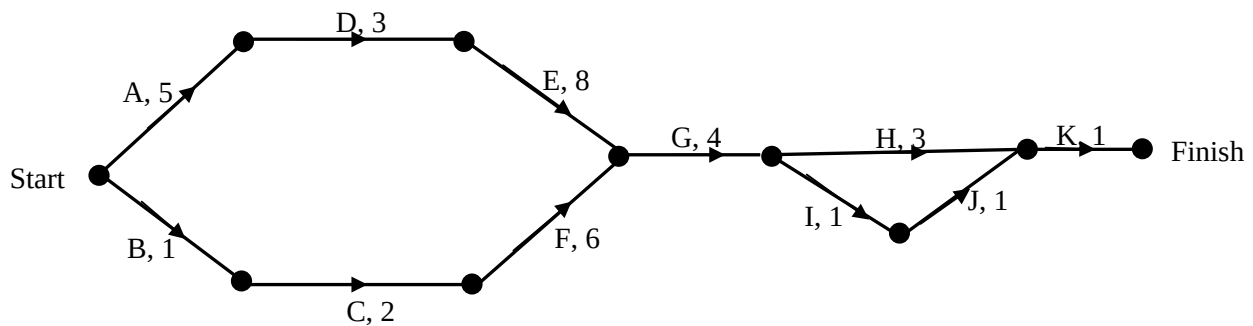
Question 3

Brooke wants to do some home renovations. She wants a new kitchen and to have wooden floorboards installed in her kitchen, the living rooms and the corridor.

She has listed the tasks involved and the time, in days, that she estimates each task will take.

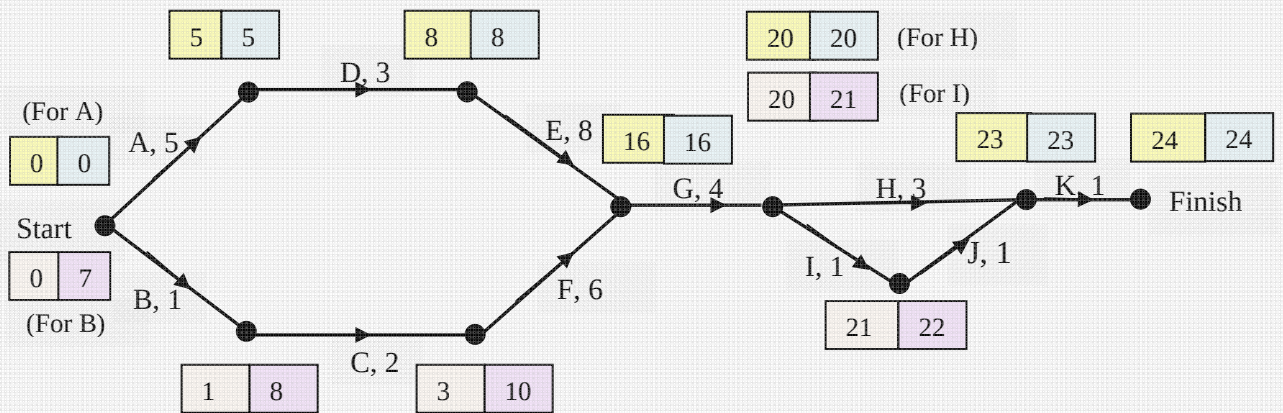
Activity	Description	Immediate predecessors	Duration of activity (days)
A	Measure the kitchen and design a new kitchen	–	5
B	Measure the area for the wooden floorboards	–	1
C	Remove the tiles	B	2
D	Remove the old kitchen	A	3
E	Order and buy the kitchen cabinets	D	8
F	Order and buy floorboards	C	6
G	Lay the wooden floorboards	E, F	4
H	Install the new kitchen cabinets	G	3
I	Sand the floorboards	G	1
J	Polish the floorboards	I	1
K	Remove the rubbish	H, J	1

Brooke has also constructed a network diagram for the project.



- a. What is the minimum number of days needed to complete this project?

Worked solution



The minimum number of days needed to complete this project is 24 days.

1 mark

- b. State the tasks that can't be delayed without delaying the whole project.

Worked solution

This is the critical path of the project: A-D-E-G-H-K.

2 marks

Mark allocation

- 1 mark for listing at least four of the six tasks correctly.
- 1 mark for the correct answer.

- c. State the float time for activity B.

Worked solution

float time = latest start time – earliest start time = 7 – 0 = 7 days

1 mark

- d. State the latest finish time for activity F.

Worked solution

latest finish time = latest start time + duration of the activity

latest finish time = 10 + 6 = 16 days

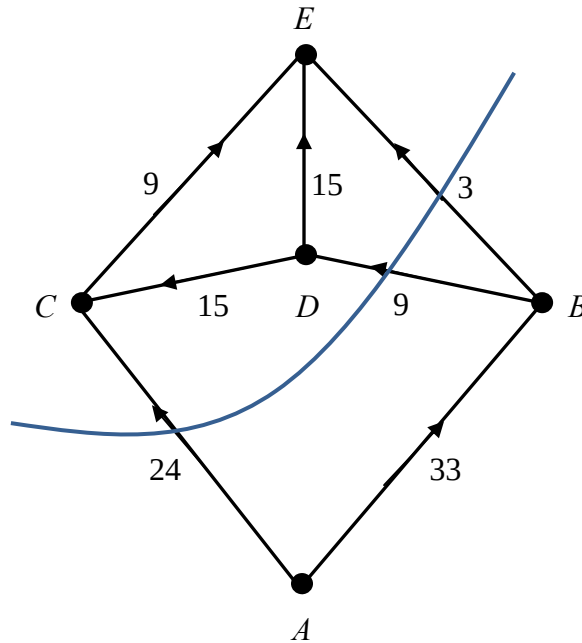
1 mark

Question 4

A school maths club organises an excursion to a puzzle and maze centre.

The rules for the giant maze are that all students must start at maze point *A* and finish at maze point *E* but not visit every point. Different students visit different points.

The maximum number of students at each of the maze points is indicated on the network below. A cut is shown on this network.



- a. What is the capacity of the cut shown in this network?

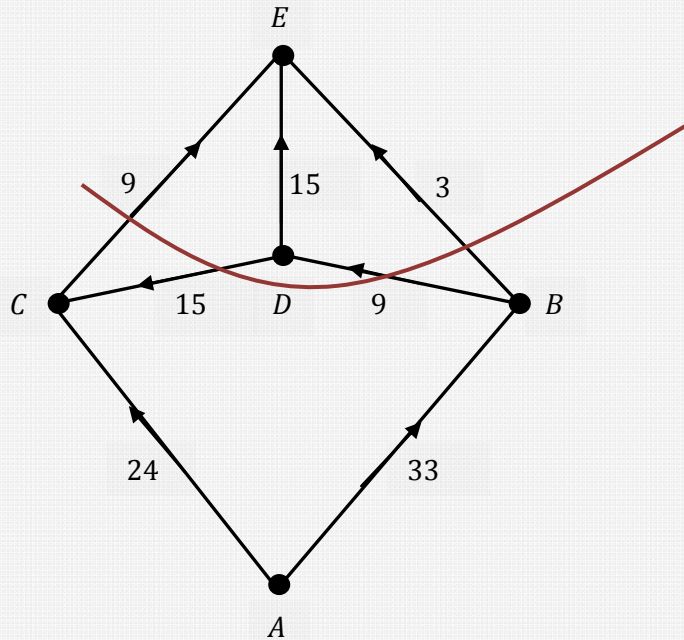
Worked solution

The capacity of the cut = $24 + 9 + 3 = 36$

1 mark

- b. What is the maximum number of students who can attend the maze activity at any one time?

Worked solution



Minimum cut = maximum flow = $9 + 0 + 9 + 3 = 21$

So a maximum number of 21 students can attend the maze activity at any one time.

1 mark



Tip

- For edge CD, the flow is from up to down of the cut illustrated. Therefore it contributes nothing to the flow from down to up.

Total 15 marks

Module 6: Matrices

Question 1

A grocery shop sells four types of rice: medium-grain rice (M), Arborio rice (A), jasmine rice (J) and basmati rice (B).

The cost of a 1 kilogram bag of each type of rice is \$2.15, \$2.75, \$3.00 and \$3.20, respectively.

- a. Write down a column matrix, C , to show the cost of a bag of each type of rice.

Worked solution

$$C = \begin{bmatrix} 2.15 \\ 2.75 \\ 3.00 \\ 3.20 \end{bmatrix} \begin{matrix} M \\ A \\ J \\ B \end{matrix}$$

1 mark

- b. What is the order of matrix C ?

Worked solution

The order of matrix C is 4×1 .

1 mark

- c. Helena purchased five bags of medium-grain rice, two bags of Arborio rice, three bags of jasmine rice and one bag of basmati rice.

Express these purchase amounts, in the order given, in a row matrix named P .

Worked solution

$$P = [5 \quad 2 \quad 3 \quad 1]$$

1 mark

- d. i.** Write down a matrix product that, when evaluated, will contain the total amount of money that Helena paid to purchase the four different types of rice.

Worked solution

$$P \times C = \begin{bmatrix} 5 & 2 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 2.15 \\ 2.75 \\ 3.00 \\ 3.20 \end{bmatrix}$$

1 mark

- ii.** Find the total cost of the rice that Helena purchased by evaluating this matrix product.

Worked solution

$$P \times C = \begin{bmatrix} 5 & 2 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 2.15 \\ 2.75 \\ 3.00 \\ 3.20 \end{bmatrix} = \begin{bmatrix} 28.45 \end{bmatrix}$$

The total cost of the rice that Helena purchased is \$28.45.

1 mark

1 + 1 = 2 marks

Question 2

The grocery shop buys three different types of cheese from three different suppliers. The amount, in kilograms, of each type of cheese that the shop buys from each of the three suppliers and the total amount of money that is paid to each supplier are given in the table below.

	Supplier A	Supplier B	Supplier C	
Mozzarella	23	25	9	
Fetta	12	28	14	
Cheddar	14	16	19	
TOTAL PRICE	\$463.60	\$643	\$404.80	

- 1 kilogram of mozzarella cheese costs \$x.
 - 1 kilogram of fetta cheese costs \$y.
 - 1 kilogram of cheddar cheese costs \$z.
- a. Three simultaneous equations were set up to find the unit prices of each type of cheese. Two of these equations are given below. Write down the third equation.

Equation 1: $23x + 12y + 14z = 463.60$

Equation 2: $25x + 28y + 16z = 643$

Equation 3: _____

Worked solution

Equation 3: $9x + 14y + 19z = 404.80$

1 mark

- b. Represent these simultaneous equations in matrix form, using the following template.

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

Worked solution

$$\begin{bmatrix} 23 & 12 & 14 \\ 25 & 28 & 16 \\ 9 & 14 & 19 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 463.60 \\ 643 \\ 404.80 \end{bmatrix}$$

1 mark

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- c. Write down an inverse matrix that can be used to solve these equations.

Worked solution

$$\begin{bmatrix} 23 & 12 & 14 \\ 25 & 28 & 16 \\ 9 & 14 & 19 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{77}{1121} & -\frac{8}{1121} & -\frac{50}{1121} \\ -\frac{331}{4484} & \frac{311}{4484} & -\frac{9}{2242} \\ \frac{49}{2242} & -\frac{107}{2242} & \frac{86}{1121} \end{bmatrix}$$

1 mark

- d. Solve the equations and, hence, evaluate the values of x , y and z .

Worked solution

$$\begin{bmatrix} 23 & 12 & 14 \\ 25 & 28 & 16 \\ 9 & 14 & 19 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 463.60 \\ 643 \\ 404.80 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23 & 12 & 14 \\ 25 & 28 & 16 \\ 9 & 14 & 19 \end{bmatrix}^{-1} \begin{bmatrix} 463.60 \\ 643 \\ 404.80 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{77}{1121} & -\frac{8}{1121} & -\frac{50}{1121} \\ -\frac{331}{4484} & \frac{311}{4484} & -\frac{9}{2242} \\ \frac{49}{2242} & -\frac{107}{2242} & \frac{86}{1121} \end{bmatrix} \begin{bmatrix} 463.60 \\ 643 \\ 404.80 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9.20 \\ 8.75 \\ 10.50 \end{bmatrix}$$

So, $x = 9.20$, $y = 8.75$ and $z = 10.50$

2 marks

Mark allocation

- 1 mark for writing the solution equation $X = A^{-1} \times C$ correctly.
- 1 mark for the correct answer.

Question 3

The grocery shop has got three branches in Melbourne. The first one is in the northern suburbs (N), the second one is in the eastern suburbs (E), and the third one is in the western suburbs (W).

The matrix C_0 displays the initial number of customers that each of the three branches had.

$$C_0 = \begin{bmatrix} 3\,000 \\ 4\,000 \\ 7\,000 \end{bmatrix} \begin{matrix} N \\ E \\ W \end{matrix}$$

- a. In total, how many customers did the shop have initially in all three branches?

Worked solution

Total number of customers = $3\,000 + 4\,000 + 7\,000 = 14\,000$ people.

1 mark

The number of customers of each shop after n years is determined using the matrix equation

$$C_n = TC_{n-1}$$

In this equation, T is the transition matrix.

$$T = \begin{matrix} \begin{matrix} & \text{this month} \\ & N & E & W \end{matrix} \\ \begin{bmatrix} 0.65 & 0.35 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.15 & 0.35 & 0.3 \end{bmatrix} \begin{matrix} N \\ E \\ W \end{matrix} & \text{next month} \end{matrix}$$

- b. i. Determine C_1

Worked solution

$$C_1 = T^1 C_0$$

$$C_1 = \begin{bmatrix} 0.65 & 0.35 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.15 & 0.35 & 0.3 \end{bmatrix} \begin{bmatrix} 3\,000 \\ 4\,000 \\ 7\,000 \end{bmatrix} = \begin{bmatrix} 4\,750 \\ 5\,300 \\ 3\,950 \end{bmatrix}$$

1 mark

- ii. Determine the expected number of customers of each shop in the long term.
Write your answer correct to the nearest whole number.

Worked solution

$$C_n = T^n C_0$$

$$C_{50} = T^{50} C_0$$

$$C_{50} = \begin{bmatrix} 0.65 & 0.35 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.15 & 0.35 & 0.3 \end{bmatrix}^{50} \begin{bmatrix} 3\,000 \\ 4\,000 \\ 7\,000 \end{bmatrix} = \begin{bmatrix} 6\,255.319\,149 \\ 4\,269.503\,546 \\ 3\,475.177\,305 \end{bmatrix}$$

So, the expected number of customers of the northern suburbs branch is 6 255, eastern suburbs branch is 4 270 and western suburbs branch is 3 475.

1 mark

1 + 1 = 2 marks

- c. After how many years will the expected number of customers of the northern suburbs branch first double?

Worked solution

$$\text{After 1 year: } C_1 = \begin{bmatrix} 0.65 & 0.35 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.15 & 0.35 & 0.3 \end{bmatrix} \begin{bmatrix} 3\,000 \\ 4\,000 \\ 7\,000 \end{bmatrix} = \begin{bmatrix} 4\,750 \\ 5\,300 \\ 3\,950 \end{bmatrix}$$

$$\text{After 2 years: } C_2 = \begin{bmatrix} 0.65 & 0.35 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.15 & 0.35 & 0.3 \end{bmatrix}^2 \begin{bmatrix} 3\,000 \\ 4\,000 \\ 7\,000 \end{bmatrix} = \begin{bmatrix} 5\,732.5 \\ 4\,515 \\ 3\,752.5 \end{bmatrix}$$

$$\text{After 3 years: } C_3 = \begin{bmatrix} 0.65 & 0.35 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.15 & 0.35 & 0.3 \end{bmatrix}^3 \begin{bmatrix} 3\,000 \\ 4\,000 \\ 7\,000 \end{bmatrix} = \begin{bmatrix} 6\,056.875 \\ 4\,377.25 \\ 3\,565.875 \end{bmatrix}$$

So, the expected number of customers of the northern suburbs branch will first double after 3 years.

2 marks

Mark allocation

- 1 mark for calculating the expected number of customers after 1 year.
- 1 mark for the correct answer.

Total 15 marks

END OF MODULE 6

END OF SOLUTIONS BOOK