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INSIGHT
YEAR 12 Trial Exam Paper

2012
FURTHER MATHEMATICS
Written examination 2
QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour 30 minutes
Structure of book

Core		
<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
3	3	15
Module		
<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
6	3	45
Total		60

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference that may be annotated (can be typed, handwritten or a textbook), one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT have to be cleared.
- Students are NOT permitted to bring blank sheets of paper or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 39 pages.
- A separate sheet with miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, diagrams in this book are **not** drawn to scale.

At the end of the examination

- You may keep this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected. You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , surds or fractions. Diagrams are not to scale unless specified otherwise.

	Page
Core	3
 Module	
Module 1: Number patterns	8
Module 2: Geometry and trigonometry	13
Module 3: Graphs and relations	19
Module 4: Business-related mathematics	25
Module 5: Networks and decision mathematics	30
Module 6: Matrices	35

Core

Question 1

The school-assessed coursework results that 28 year 12 Further Mathematics students received from two different analysis tasks are given in the back-to-back stemplot shown in Figure 1 below.

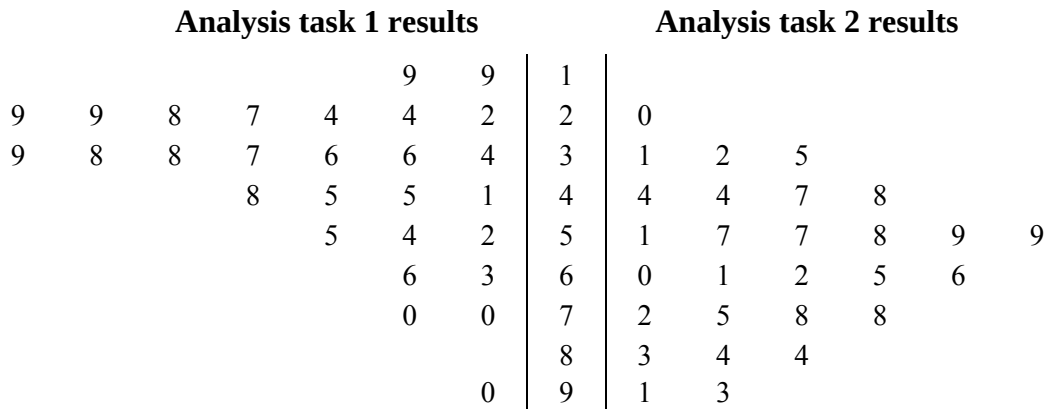


Figure 1

- a. For these 28 students, determine
- i. the median of the analysis task 1 results.

- ii. the interquartile range of the analysis task 1 results.

1 + 1 = 2 marks

- b. If the data values representing analysis task 1 results displayed in Figure 1 were used to construct a boxplot with outliers, then would the student who received 90 be shown as an outlier?

Show an appropriate calculation to support your answer.

2 marks

- c. Both the median and the mean are appropriate measures of centre of the distribution of the analysis task 2 results.

Explain why.

1 mark

Question 2

The average monthly incomes (in dollars) and the average monthly expenditures (in dollars) for a group of 15 households have been recorded. The results are displayed in Table 1 below.

Average monthly income (\$)	Average monthly expenditure (\$)
3 000	2 900
1 500	2 500
5 100	3 200
5 200	4 700
3 400	2 500
7 500	6 000
10 250	11 000
2 500	2 350
4 200	4 000
6 500	5 800
8 250	7 250
12 300	10 750
4 200	4 100
2 700	2 000
8 900	9 200

Table 1

The aim is to find a linear equation that allows average monthly expenditure to be predicted from average monthly income.

- a. What will be the dependent variable in the equation?

1 mark

- b. The equation of the least squares regression is to be calculated for this set of data. It has been assumed therefore that the relationship between average monthly income and average monthly expenditure is _____.

1 mark

- c. Determine the equation of the least squares regression line that enables *average monthly expenditure* to be predicted from *average monthly income*.
Write this equation in terms of the variables *average monthly expenditure* and *average monthly income*.
Give the coefficients, correct to two decimal places.

2 marks

- d. Using the equation that you have determined in **part c**, interpret the slope of the least squares regression line in terms of the variables *average monthly income* and *average monthly expenditure*.

1 mark

- e. Determine and interpret the coefficient of determination in terms of the variables *average monthly income* and *average monthly expenditure*.
Write your answer as a percentage, correct to two decimal places.

1 mark

- f. i.** Use the least squares regression line equation that you have determined in **part c** to predict the average monthly expenditure (in dollars) of a household for which the average monthly income is \$20 000.

Give your answer correct to the nearest hundred dollars.

- ii.** The prediction made in **part f i** is not likely to be reliable.
Explain why.

1 + 1 = 2 marks

Question 3

Table 2 shows the seasonal indices for ice-cream sales in summer, autumn, winter and spring.

Seasonal indices			
Summer	Autumn	Winter	Spring
1.525	0.755	0.625	1.095

Table 2

- a.** By what percentage should the ice-cream sales figures for winter be increased in order to correct for seasonality?

1 mark

- b.** What does a seasonal index of 1.525 tell us about the ice-cream sales in summer?

1 mark

Total 15 marks

END OF CORE

**END OF CORE
TURN OVER**

Module 1: Number patterns

Question 1

Susan received 34 out of 100 for her first English assignment in the 2012 academic year. She submitted all her assignments regularly and studied hard to improve her results. Consequently, she increased her mark by 6 in each of the subsequent assignments.

- a. What type of sequence is formed by Susan's assignment marks?

1 mark

- b. What mark did Susan receive for her fifth English assignment?

1 mark

- c. Susan received 100 for the last English assignment of the year.
How many English assignments did she have during 2012?

1 mark

- d. Find the average mark of Susan's English assignments for the year 2012.

1 mark

Question 2

When Susan's school first opened it had 240 students. The number of students enrolled at this school increased by 20% each subsequent year.

a. Determine the number of students the school had

i. after 1 year of opening.

ii. after 3 years of opening.

Write your answer correct to the nearest whole number.

1 + 1 = 2 marks

b. Write an expression that gives the number of students, K_n , the school has after n years of opening.

1 mark

c. How many more students does the school have after 7 years than after 6 years of opening?

Write your answer correct to the nearest whole number.

1 mark

- d.** After how many years will the school first have more than 1000 students?

1 mark

When Susan's school first opened it had 14 staff members. At the start of each academic year three more staff members are recruited to meet the increasing demand of the students.

A staff photo is taken at the start of each academic year. This photo is then placed on a pin board that has staff photos for all the previous years, in order by year.

- e.** Susan's brother Jamie counts the total number of staff members in the first five photos to be 100. He then gets tired and asks Susan to continue counting the total number of staff members in the next five photos.

Assuming that neither Jamie nor Susan make counting errors, how many staff members does Susan count in the next five photos?

1 mark

Question 3

The yearly school tuition fee, in dollars, is generated by the difference equation

$$F_{n+1} = 0.9F_n + 2\,250 \quad F_0 = 15\,000$$

Where F_n is the yearly school tuition fee n years after the school's opening.

- a.** Show that the difference equation is not geometric.

1 mark

- b.** Use the difference equation to determine the yearly school tuition fee for a student 3 years after the school's opening.

Write your answer correct to the nearest cent.

1 mark

- c.** After how many years will the yearly tuition fee of the school exceed \$20 000?

1 mark

- d.** What is the total amount of school tuition fees that the school collected in the first 2 years?

2 marks

Total 15 marks

Module 2: Geometry and trigonometry

Question 1

Juliet is an interior designer and her new project is painting the interior walls of an arts centre.

The centre has a triangular shape, as shown in Figure 1 below.

The wall AB runs in an east–west line and is 56 metres long.

The wall BC is 45 metres long.

The angle between these two walls is 42° .

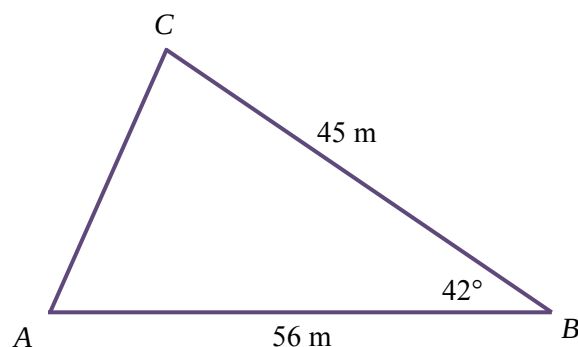


Figure 1

- a.** Find the length of the wall AC .

Write your answer in metres, correct to two decimal places.

1 mark

- b.** Using your answer from question **1a**, find the magnitude of the angle BAC .

Express your answer correct to the nearest degree.

2 marks

- c. Find the area of the arts centre, correct to the nearest square metre.

1 mark

Question 2

When she had finishing painting the walls, Juliet divided the arts centre into two sections, as shown in Figure 2 below.

She then started decorating section 2 with Aboriginal motifs.

The length of the wall DB is 33.44 metres and the new wall CD runs in a north–south line.

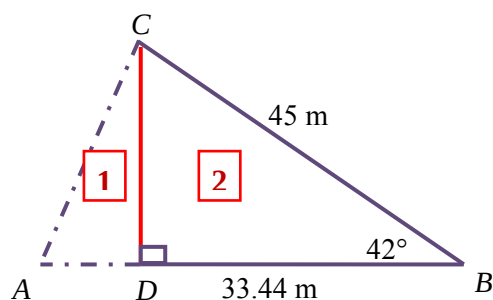


Figure 2

- a. Find the bearing of D from C .

1 mark

- b. Find the length of the new wall CD , correct to the nearest metre.

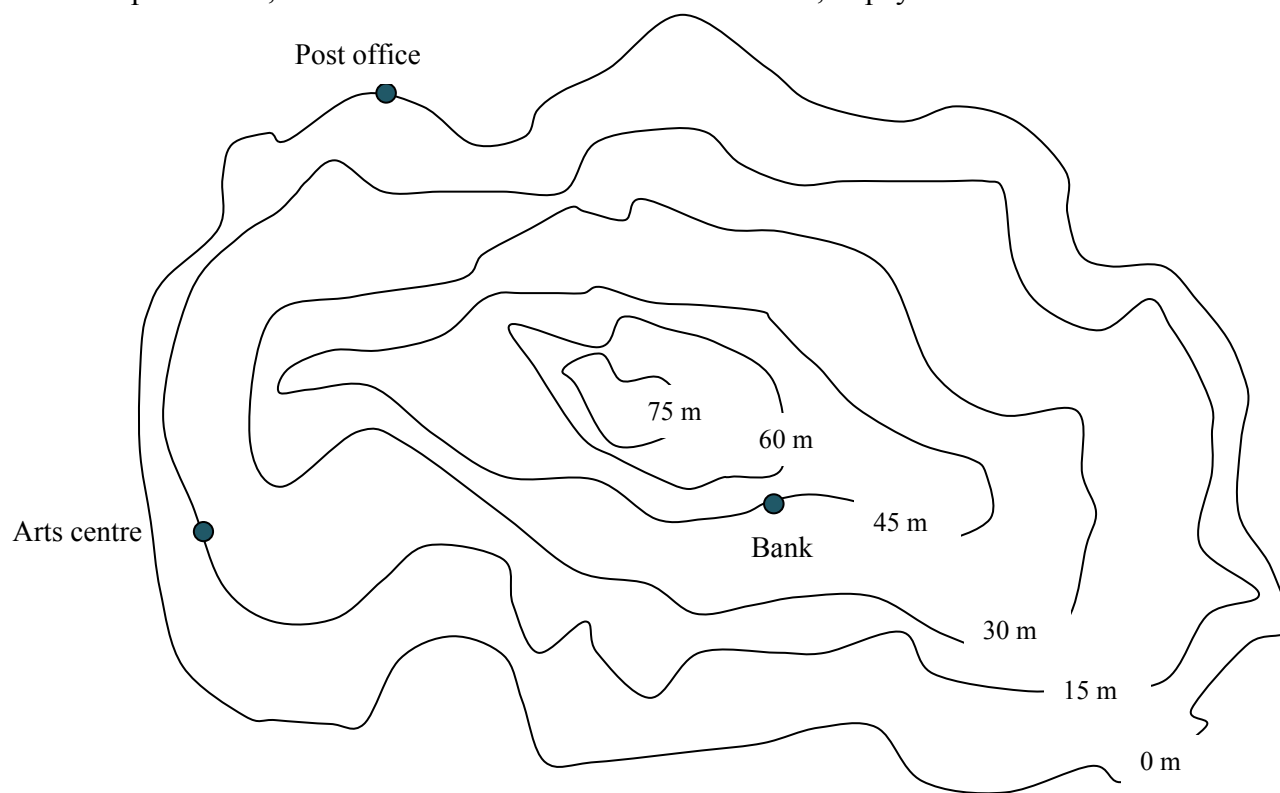
1 mark

- c. Juliet made a scale diagram of section 2 to draw better plans for decorating.
The ratio of the area of section 2 on Juliet's diagram to the actual area is 1:121.
What was the length of the wall DB on Juliet's diagram?
Write your answer in centimetres.

2 marks

Question 3

The arts centre is located at an altitude of 15 metres on a hillside. A contour map of the hill is shown in Figure 3 below. During a lunch break while working on her project, Juliet goes to the bank, which is located at an altitude of 45 metres, to withdraw money. She then goes to the post office, which is located at an altitude of 0 metres, to pay her bills.

**Figure 3**

On the contour map, 1 cm represents 10 metres on the horizontal level.

- a.** Determine the vertical distance between the arts centre and the bank.

1 mark

- b.** The map distance between the arts centre and the bank is given as 9 centimetres. Determine the actual distance, in metres, from the arts centre to the bank. Write your answer in metres, correct to three decimal places.

2 marks

- c.** The horizontal distance between the bank and the post office is 72 metres. Calculate the average slope between the bank and the post office.

1 mark

- d.** Map distance between the post office and the arts centre is given as 11.5 centimetres. Calculate the angle of depression of a person looking at the post office from the arts centre. Write your answer in degrees, correct to two decimal places.

1 mark

- e.** On another day Juliet goes to the bank and then to the market.
The market is located at an altitude of 75 metres.
The angle of elevation of the market from the bank is 45° .
Determine the horizontal distance and the actual distance between the bank and the market.
Write your answers in metres, correct to one decimal place.

2 marks

Total 15 marks

Module 3: Graphs and relations

Question 1

The towns of Lorne and Apollo Bay are 45 km apart. A cyclist, Margaret, leaves Lorne and travels towards Apollo Bay.

- a.** Margaret cycles at a constant speed of 9 km/h for the first 3 hours without taking any rest periods. How far does she travel in the first 3 hours of her journey?

1 mark

- b.** After cycling for 3 hours, Margaret decides to stop for a 30 minute lunch break. She then cycles at a constant speed of 12 km/h for the rest of her journey.
How long does her journey from Lorne to Apollo Bay take?

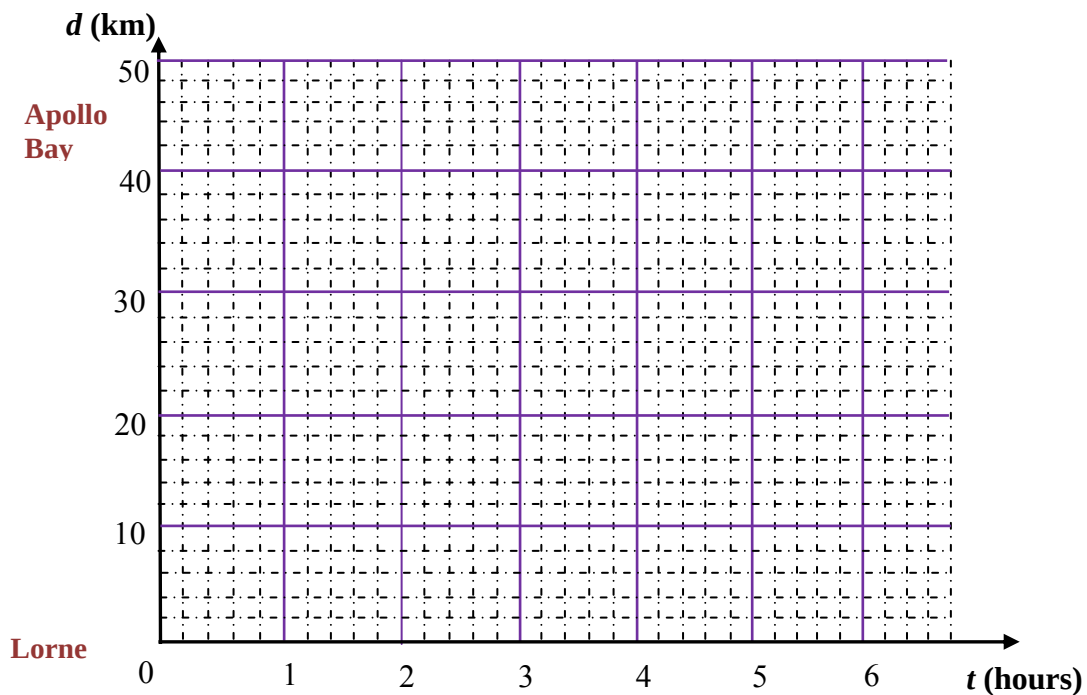
1 mark

- c.** What is Margaret's average speed, in kilometres per hour, during her journey from Lorne to Apollo Bay?

1 mark

- d. Let d represent the distance cycled by Margaret and t the time taken during her journey.

Draw and label the graph of d (km) against t (hours) on the set of axes below.



1 mark

Another cyclist, Richard, leaves Apollo Bay and travels towards Lorne. Richard and Margaret start their journeys towards each other at the same time.

The remaining distance of Richard from Lorne, in kilometres, is given by the equation $d = 45 - 7.5t$ for $0 \leq t < k$, where t is the time, in hours, since he began cycling.

- e. Determine the value of k .

1 mark

- f.** After how many hours did their paths cross?
Write your answer correct to one decimal place.

1 mark

- g.** Where did they pass each other?
Write your answer correct to one decimal place.

1 mark

Question 2

A skip bin hire company has two different types of skip bins for hire: small bins and large bins.

Let x be the number of small skip bins that are hired each day.

Let y be the number of large skip bins that are hired each day.

A constraint on the number of skip bins that can be hired each day is given by

Inequality 1: $x + y \leq 7$

- a.** Explain the meaning of Inequality 1 in terms of the context of this problem.

1 mark

Each small bin requires four people to deliver it and each large bin requires five people.

There are only five small bins and three large bins available to hire each day.

There are only 24 people available to do the delivery of the skip bins each day.

These constraints are given by

Inequality 2: $4x + 5y \leq 24$

Inequality 3: $x \geq 0$

Inequality 4: $y \geq 0$

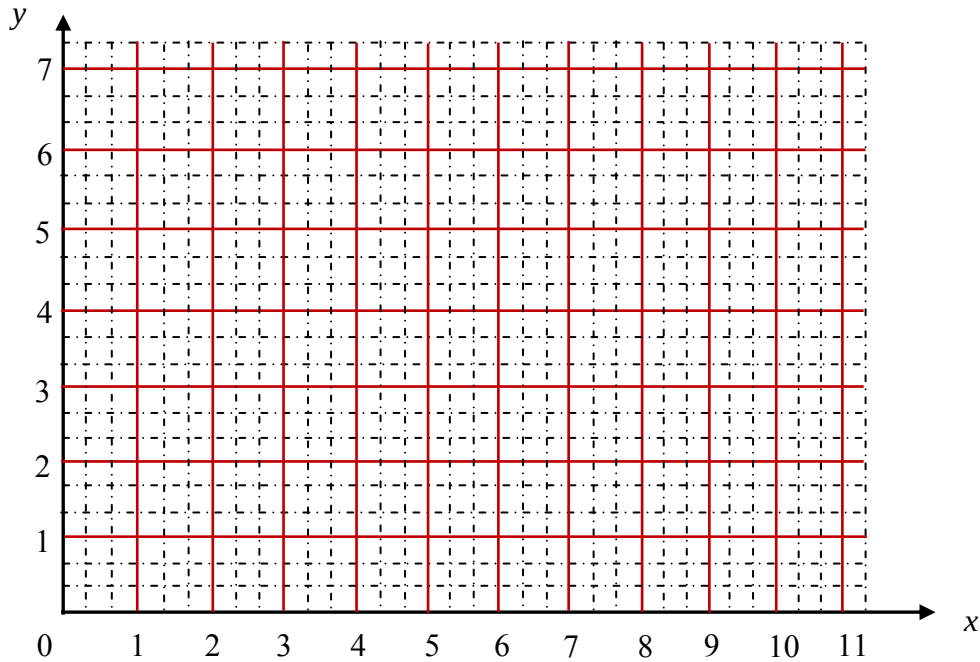
Inequality 5: $x \leq 5$

Inequality 6:

- b.** Fill in the sixth constraint on the number of the large skip bins that can be hired each day.

1 mark

- c. Draw the line that represents $4x + 5y = 24$ on the graph below.



1 mark

- d. i. On the graph given in **part c**, draw and clearly indicate the boundaries of the region represented by inequalities 1 to 6. Shade the feasible region.
- ii. On a day when exactly two large skip bins are hired, what is the maximum number of small bins that could be hired?

2 + 1 = 3 marks

The profit from a small skip bin hire is \$70 and the profit from a large skip bin hire is \$110.
Let P be the total profit obtained from small and large skip bin hires.

- e. Write an equation for the total profit, P , in terms of x and y .

1 mark

- f. Determine the maximum total profit that the skip bin hire company can make per day from the hire of these skip bins.

1 mark

Total 15 marks

Module 4: Business-related mathematics

Question 1

Jessica invests \$100 000 into a bank account that earns a compound interest of 5.2%, compounding annually. She does not make any deposits or withdrawals for 20 years.

- a.** Determine the total value of this investment at the end of 20 years.

Write your answer correct to the nearest dollar.

1 mark

- b.** If Jessica had invested the \$100 000 in an account that earned 5.2% per annum simple interest, what would the investment have been worth after 20 years?

1 mark

- c.** If Jessica had invested the \$100 000 in an account that earned 5.2% per annum compound interest, compounding monthly, what would the investment have been worth after 20 years?

Write your answer correct to the nearest cent.

1 mark

- d.** Jessica's friend Monica has also invested \$100 000 into an account that earns a compound interest of 5.2% per annum, compounding quarterly, and makes no further deposits or withdrawals to it.

If Monica invested her money into this account at the start of January 2012, when would the amount in this account first be more than double?

2 marks

Question 2

Jessica's brother, Ben, takes out a reducing balance loan of \$750 000 at an interest rate of 7.4% per annum, compounding monthly, to buy a new house.

- a.** Ben is planning to repay his loan in full in 10 years.
- i.** What is the monthly repayment he needs to pay?
Write your answer correct to the nearest cent.

- ii.** Calculate the total interest that Ben will pay over the 10-year period.
Write your answer correct to the nearest dollar.

1 + 1 = 2 marks

- b.** Suppose, instead, Ben pays \$6 000 each month for the first 6 years of the loan and then he increases his payments to \$7 000 per month for the following 4 years.
Find the amount outstanding on the loan at the end of the 10-year period.
Write your answer correct to the nearest cent.

2 marks

Question 3

Ben purchases a car with a discounted price of \$35 000.

- a.** The original price of the car before discount was \$40 000. What is the percentage discount that Ben received?

1 mark

- b.** Ben agrees to a hire-purchase contract for the car costing \$35 000, which requires a deposit of \$15 000 and 60 equal monthly payments of \$550.

Find the effective rate of interest per annum that he is paying for this contract.

Express your answer as a percentage, correct to two decimal places.

1 mark

- c.** The car that Ben purchases for \$35 000 is to be depreciated over time for taxation purposes. If the car is depreciated by 20 cents per kilometre, how many kilometres will the car travel before its book value is reduced to \$20 000?

1 mark

- d.** If the car is depreciated by 5% of its purchase price each year, what is the book value of the car after 3 years?

1 mark

- e.** If the value of the car depreciates at a rate of 6% per annum, by how much will the car depreciate during the fifth year?

Write your answer correct to the nearest dollar.

2 marks

Total 15 marks

END OF MODULE 4
TURN OVER

Module 5: Networks and decision mathematics

Question 1

Josh, an information technology technician, needs to visit nine houses on Monday. The network below shows the roads that are used to travel between the houses, which are labelled from M to V .

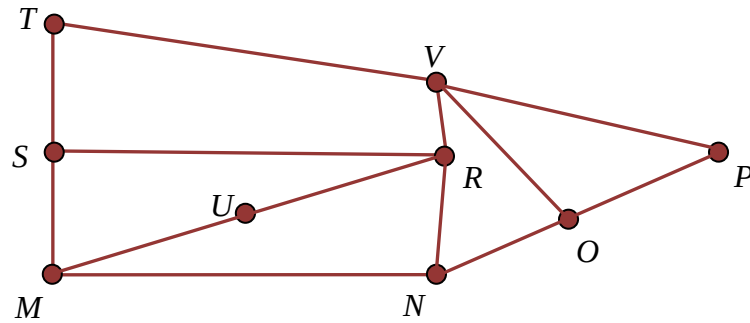


Figure 1

- a. Find the sum of the degrees of the vertices of the network shown in Figure 1.

1 mark

- b. Write down a Hamiltonian circuit that starts and finishes at U .

1 mark

- c. Verify that the graph has six faces, using Euler's formula.

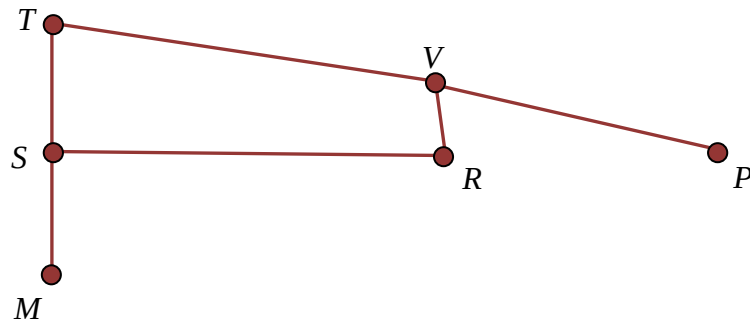
1 mark

- d. Josh wants to start at point M and travel each road exactly once, finishing at point M . He realises that this is impossible. Explain why such a circuit does not exist.

2 marks

Question 2

On Tuesday, Josh gets sick and sends his partner Gary to complete the jobs with the highest priority. These houses and the roads linking them are shown in Figure 2.

**Figure 2**

The adjacency matrix below indicates the houses that have a road running directly between them.

	M	P	R	S	T	V
M	0	0	0	1	0	0
P	0	0	0	0	0	1
R	a	0	0	1	0	1
S	1	0	b	0	1	0
T	0	0	0	1	0	c
V	0	1	1	0	1	0

- a. Explain why $c = 1$.

1 mark

- b. Find the sum of the values a and b in the adjacency matrix above.

1 mark

- c. What is the sum of the elements in column 4 of the adjacency matrix and what does this tell us in the context of this problem?

1 mark

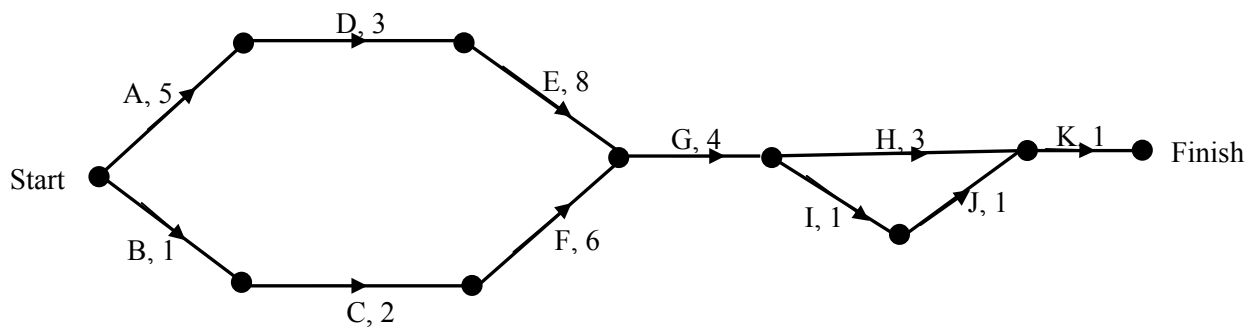
Question 3

Brooke wants to do some home renovations. She wants a new kitchen and to have wooden floorboards installed in her kitchen, the living rooms and the corridor.

She has listed the tasks involved and the time, in days, that she estimates each task will take:

Activity	Description	Immediate predecessors	Duration of activity (days)
A	Measure the kitchen and design a new kitchen	–	5
B	Measure the area for the wooden floorboards	–	1
C	Remove the tiles	B	2
D	Remove the old kitchen	A	3
E	Order and buy the kitchen cabinets	D	8
F	Order and buy floorboards	C	6
G	Lay the wooden floorboards	E, F	4
H	Install the new kitchen cabinets	G	3
I	Sand the floorboards	G	1
J	Polish the floorboards	I	1
K	Remove the rubbish	H, J	1

Brooke has also constructed a network diagram for the project.



- a.** What is the minimum number of days needed to complete this project?

1 mark

- b.** State the tasks that can't be delayed without delaying the whole project.

2 marks

- c.** State the float time for activity B.

1 mark

- d.** State the latest finish time for activity F.

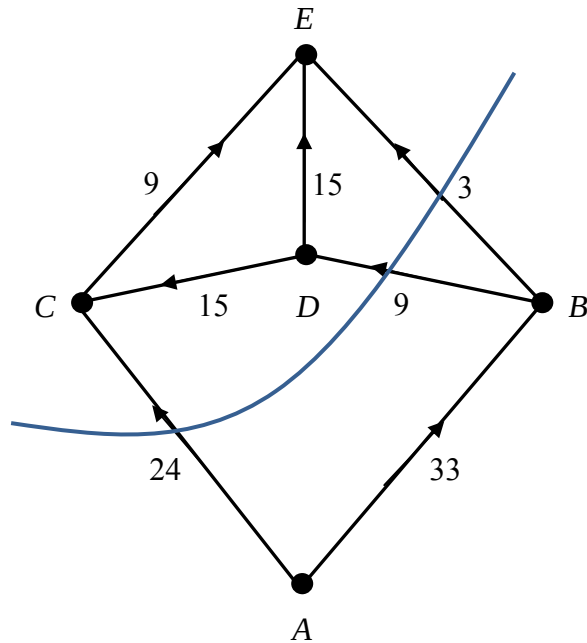
1 mark

Question 4

A school maths club organises an excursion to a puzzle and maze centre.

The rules for the giant maze are that all students must start at maze point *A* and finish at maze point *E* but not visit every point. Different students visit different points.

The maximum number of students at each of the maze points is indicated on the network below. A cut is shown on this network.



- a. What is the capacity of the cut shown in this network?

1 mark

- b. What is the maximum number of students who can attend the maze activity at any one time?

1 mark

Total 15 marks

Module 6: Matrices

Question 1

A grocery shop sells four types of rice: medium-grain rice (M), Arborio rice (A), jasmine rice (J) and basmati rice (B).

The cost of a 1 kilogram bag of each type of rice is \$2.15, \$2.75, \$3.00 and \$3.20, respectively.

- a. Write down a column matrix, C , to show the cost of a bag of each type of rice.

1 mark

- b. What is the order of matrix C ?

1 mark

- c. Helena purchased five bags of medium-grain rice, two bags of Arborio rice, three bags of jasmine rice and one bag of basmati rice.

Express these purchase amounts, in the order given, in a row matrix named P .

1 mark

- d. i. Write down a matrix product that, when evaluated, will contain the total amount of money that Helena paid to purchase the four different types of rice.

- ii. Find the total cost of the rice that Helena purchased by evaluating this matrix product.

1 + 1 = 2 marks

Module 6: Matrices – continued
TURN OVER

Question 2

The grocery shop buys three different types of cheese from three different suppliers. The amount, in kilograms, of each type of cheese that the shop buys from each of the three suppliers and the total amount of money that is paid to each supplier are given in the table below.

	Supplier A	Supplier B	Supplier C	
Mozzarella	23	25	9	
Fetta	12	28	14	
Cheddar	14	16	19	
TOTAL PRICE	\$463.60	\$643	\$404.80	

- 1 kilogram of mozzarella cheese costs \$x.
- 1 kilogram of fetta cheese costs \$y.
- 1 kilogram of cheddar cheese costs \$z.

- a. Three simultaneous equations were set up to find the unit prices of each type of cheese. Two of these equations are given below. Write down the third equation.

Equation 1: $23x + 12y + 14z = 463.60$

Equation 2: $25x + 28y + 16z = 643$

Equation 3: _____

1 mark

- b. Represent these simultaneous equations in matrix form, using the following template.

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

1 mark

- c. Write down an inverse matrix that can be used to solve these equations.

1 mark

- d. Solve the equations and, hence, evaluate the values of x , y and z .

2 marks

Question 3

The grocery shop has got three branches in Melbourne. The first one is in the northern suburbs (N), the second one is in the eastern suburbs (E), and the third one is in the western suburbs (W).

The matrix C_0 displays the initial number of customers that each of the three branches had.

$$C_0 = \begin{bmatrix} 3\,000 \\ 4\,000 \\ 7\,000 \end{bmatrix} \begin{matrix} N \\ E \\ W \end{matrix}$$

- a. In total, how many customers did the shop have initially in all three branches?

1 mark

The number of customers of each shop after n years is determined using the matrix equation

$$C_n = TC_{n-1}$$

In this equation, T is the transition matrix.

$$T = \begin{matrix} & \begin{matrix} \text{this month} \\ N & E & W \end{matrix} \\ \begin{matrix} \begin{bmatrix} 0.65 & 0.35 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.15 & 0.35 & 0.3 \end{bmatrix} \begin{matrix} N \\ E \\ W \end{matrix} \\ \text{next month} \end{matrix}$$

- b. i.** Determine C_1 .
- ii.** Determine the expected number of customers of each shop in the long term.
Write your answer correct to the nearest whole number.

1 + 1 = 2 marks

- c.** After how many years will the expected number of customers of the northern suburbs branch first double?

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK