

**THE
HEFFERNAN
GROUP**

P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
Fax 03 9836 5025
info@theheffernangroup.com.au
www.theheffernangroup.com.au

Student Name.....

FURTHER MATHEMATICS

TRIAL EXAMINATION 2

2018

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains 7 short-answer and extended answer questions from the core.
Section A is compulsory and is worth 36 marks.
Section B begins on page 12 and consists of 4 modules. You should choose 2 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 12 marks.
Section B is worth 24 marks.
There are a total of 60 marks available for this exam.
The marks allocated to each of the questions are indicated throughout.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
Unless otherwise stated, the diagrams in this exam are not drawn to scale.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Formula sheets can be found on pages 28 and 29 of this exam.

This paper has been prepared independently of the Victorian Curriculum and Assessment Authority to provide additional exam preparation for students. Although references have been reproduced with permission of the Victorian Curriculum and Assessment Authority, the publication is in no way connected with or endorsed by the Victorian Curriculum and Assessment Authority.

© THE HEFFERNAN GROUP 2018

This Trial Exam is licensed on a non transferable basis to the purchasing school. It may be copied by the school which has purchased it. This license does not permit distribution or copying of this Trial Exam by any other party.

SECTION A - Core

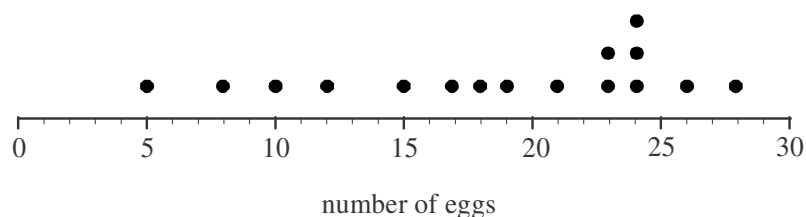
Data analysis

This section is compulsory.

Question 1 (5 marks)

Bess keeps 16 hens on her property.

The dot plot below shows the distribution of the number of eggs laid by these hens during January.



a. Write down the

i. median.

1 mark

ii. third quartile (Q_3).

1 mark

b. Find the percentage of hens which produced less than 15 eggs during January.

1 mark

A free-range egg producer has a large population of hens. The number of eggs produced in January by these hens is normally distributed with a mean of 21 eggs and a standard deviation of 2 eggs.

c. Use the 68 – 95 – 99.7% rule to determine the percentage of these hens which laid more than 25 eggs in January.

1 mark

d. The standardized number of eggs laid by one of the hens in January is given by $z = -1.5$. How many eggs did this hen lay in January?

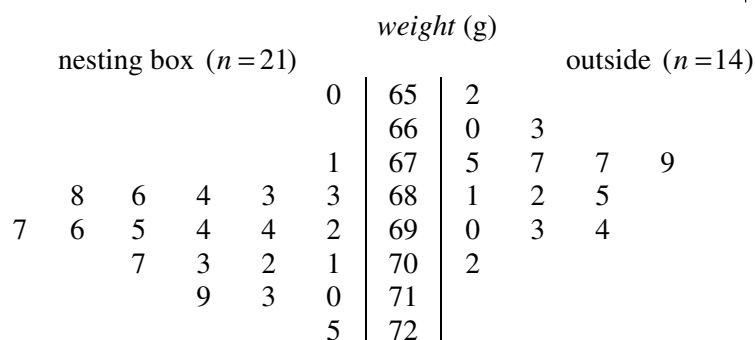
1 mark

Question 2 (7 marks)

The *weight*, in grams, and the *collection place* (nesting box or outside) of 35 eggs was recorded by a farm worker.

The back-to-back stem plot below displays the data.

key: $65|2 = 65.2$ g



- a. Find the modal weight, in grams, of the eggs collected from outside. 1 mark

- b. The five-number summary for the distribution of the weight of eggs collected from the nesting box is displayed in the table below.
The table is incomplete.

	<i>weight</i> (g)
Minimum	65.0
Q_1	68.5
Median	69.5
Q_3	70.5
Maximum	

Write down the

- i. range. 1 mark

- ii. interquartile range. 1 mark

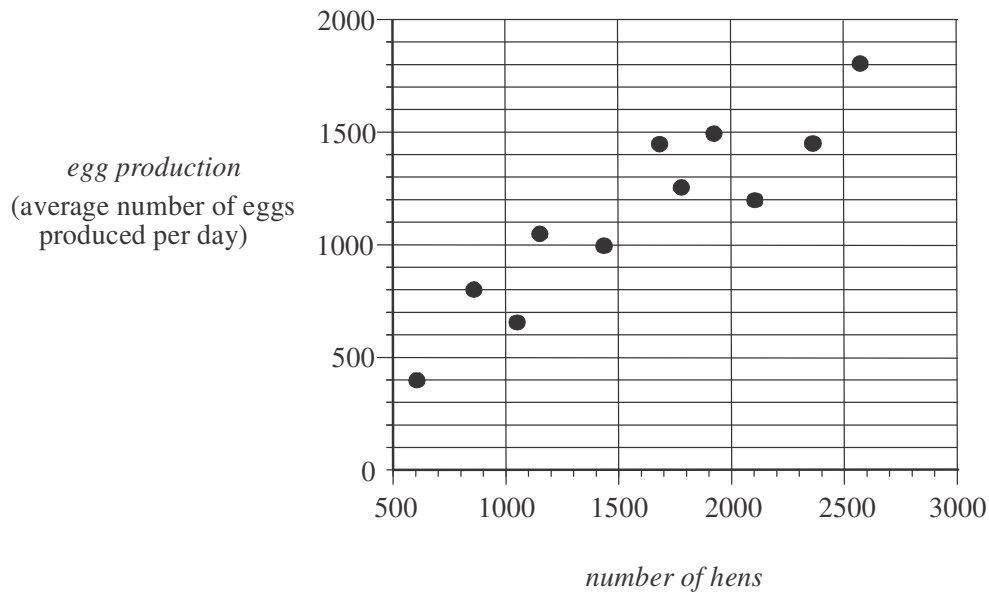
- c. Show that the egg collected from the nesting box with a weight of 65.0 g is an outlier. 2 marks

- d.** Explain why the *weight* of the collected eggs is associated with the *collection place* of the eggs. Use an appropriate statistic to support your explanation. 2 marks

Question 3 (8 marks)

The *number of hens* kept at 11 free range egg farms and the *egg production* (average number of eggs produced per day) at each of these farms is recorded.

The scatterplot below displays this data.



The equation of the least squares line is

$$\text{egg production} = 194 + 0.6 \times \text{number of hens}.$$

- a. What is the response variable? 1 mark

- b. Draw the graph of the least squares line on the scatterplot. 1 mark

- c. Interpret the slope of the least squares line in terms of the variables *egg production* and *number of hens*. 1 mark

- d. At the farm where the *number of hens* is 2 100, there is an *egg production* of 1 200 per day on average.
How many eggs does the least squares line overestimate the egg production per day for this farm? 1 mark

- e. The least squares line equation is used to predict the *egg production* at a farm where the *number of hens* is 4 500.
Give a reason why this prediction might be unreliable. 1 mark

- f. Pearson's correlation coefficient for this data is 0.9182.
Determine the percentage of the variation in *egg production* that can be explained by the variation in the *number of hens* kept at these 11 farms.
Round your answer to two decimal places. 1 mark

The *number of hens* kept at nine **non free-range** egg farms and the *egg production* (average number of eggs produced per day) at each of these farms is recorded.
The data is shown in the table below.

<i>number of hens</i>	1 820	2 450	1 950	2 130	2 210	2 040	1 980	2 380	2 010
<i>egg production</i>	1 440	1 920	1 510	1 630	1 690	1 580	1 560	1 820	1 550

- g. Determine the equation of the least squares line that can be used to predict *egg production* from the *number of hens* kept at a farm.
Write the values of the intercept and slope of the line in the boxes below. Round these values to two decimal places. 2 marks

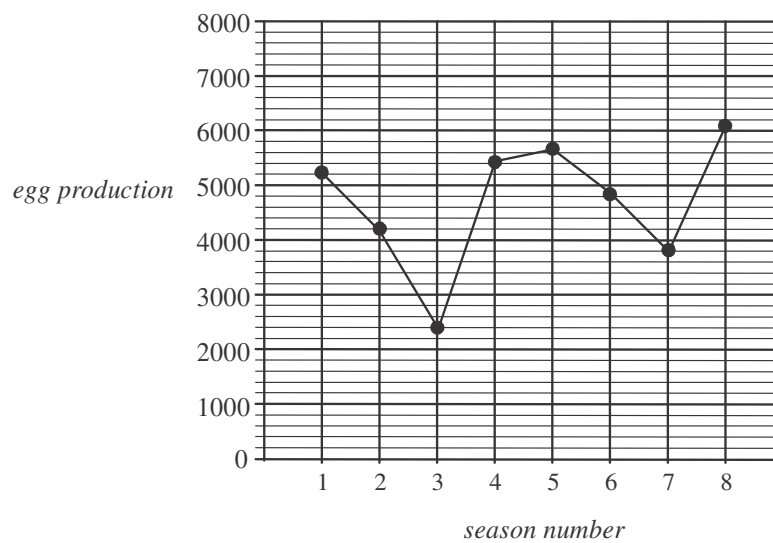
$$\text{egg production} = \boxed{} + \boxed{} \times \text{number of hens}$$

Question 4 (4 marks)

The table below shows the seasonal *egg production* at a farm for 2016 and 2017.

<i>Season</i>	Summer 2016	Autumn 2016	Winter 2016	Spring 2016	Summer 2017	Autumn 2017	Winter 2017	Spring 2017
<i>Season number</i>	1	2	3	4	5	6	7	8
<i>Egg production</i>	5290	4240	2400	5420	5680	4840	3810	6100

This data has been displayed in the time series plot below showing *egg production* against *season number*.



- a. Describe the pattern in the time series plot.

1 mark

- b. Four-mean smoothing with centring is used to smooth the time series plot above. Using the data given in the table, find the four-mean smoothed egg production centred on Summer 2017.

2 marks

The table below shows the long-term average of the seasonal egg production at the farm. The seasonal indices for Summer and Autumn are also shown.

	Season			
	Summer	Autumn	Winter	Spring
Long-term average	5 100	4 380	3 260	5 850
Seasonal index	1.10	0.94		

- c. Find the seasonal index for Winter.
Express your answer correct to two significant figures.

1 mark

Recursion and financial modelling

Question 5 (4 marks)

Jacinta is an optometrist.

The value of the equipment used at her business is depreciated using the flat rate method of depreciation.

The value of the equipment V_n , in dollars, after n years, can be modelled by the recurrence relation

$$V_0 = 54\,000, \quad V_{n+1} = V_n - 4\,590$$

- a.** How much is the equipment depreciated by each year? 1 mark

- b.** Using recursion, write down calculations that show that the value of the equipment after two years is \$44 820. 1 mark

- c.** Find the annual flat rate of depreciation of the value of equipment. Express your answer as a percentage. 1 mark

- d.** The value of the equipment at the practice could also be depreciated using the reducing balance method of depreciation at the annual percentage rate of 7.5% per year. The recurrence relation below models this but is incomplete.

$$V_0 = 54\,000, \quad V_{n+1} = \boxed{} \times V_n$$

- Complete the recurrence relation above by entering a value in the box. 1 mark

Question 6 (4 marks)

Whilst starting her business, Jacinta was unable to make a monthly payment on her credit card debt of \$5 600.

She was charged interest at the rate of 1.8% per month on the amount due plus a ‘no payment’ fee of \$15 per month for the month that she made no payment.

Jacinta made no further purchases using her credit card and paid the card off completely when she received her next statement the following month.

- a.** How much did Jacinta pay? 1 mark

Consider the situation where Jacinta made no monthly payments on her original credit card debt of \$5 600 and made no further purchases using her credit card.

Let D_n be Jacinta’s credit card debt, in dollars, n months after her initial monthly payment was due.

- b.** Write down a recurrence relation in terms of D_0 , D_{n+1} and D_n that models this debt. 2 marks

- c.** After how many months would the debt first exceed \$7 000? 1 mark

Question 7 (4 marks)

Once the business was established, Jacinta invested \$80 000 in an annuity investment. This annuity investment earns interest at the rate of 4.2% per annum compounding quarterly and each quarter Jacinta makes a further payment of \$5 000 to the investment immediately after the interest is added.

- a.** Find the value of Jacinta's investment after five years. 1 mark

- b.** How much interest did Jacinta's investment earn during these five years? 1 mark

- c.** Jacinta wants the value of her annuity investment to reach \$300 000 in just a further two years.
What will Jacinta's new quarterly payment to the investment need to be to achieve this? 1 mark

- d.** Two years later, Jacinta invests the \$300 000 from her annuity in a perpetuity for her brother. This perpetuity earns interest at the rate of 3.9% per annum with interest calculated and paid weekly.
What weekly payment will Jacinta's brother receive from this perpetuity? 1 mark

SECTION B - Modules

Module 1 - Matrices

If you choose this module all questions must be answered.

Question 1 (4 marks)

A fashion retailer sells accessories (A), boots (B) and clothes (C) at one of its stores. The number of each type of item sold at this store over a three month period is given by matrix F below.

$$F = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 720 & 405 & 980 \\ 708 & 420 & 950 \\ 690 & 390 & 970 \end{bmatrix} & \begin{matrix} month\ 1 \\ month\ 2 \\ month\ 3 \end{matrix} \end{matrix}$$

- a. How many accessories were sold in total at this store over this three-month period. 1 mark

- b. For the matrix equation,

$$\begin{bmatrix} 720 & 405 & 980 \\ 708 & 420 & 950 \\ 690 & 390 & 970 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 67\ 375 \\ 66\ 970 \\ 65\ 750 \end{bmatrix},$$

the profit made on the sale of

- an accessory is x
- a pair of boots is y
- an item of clothing is z

- i. What is the total profit made in month 3? 1 mark

- ii. What is the profit made on the sale of a pair of boots? 1 mark

- iii. The total profit made from the sale of accessories and clothes over the three months is \$133125 as indicated by the matrix equation below

$$G \times \begin{bmatrix} 67\ 375 \\ 66\ 970 \\ 65\ 750 \end{bmatrix} = [133\ 125]$$

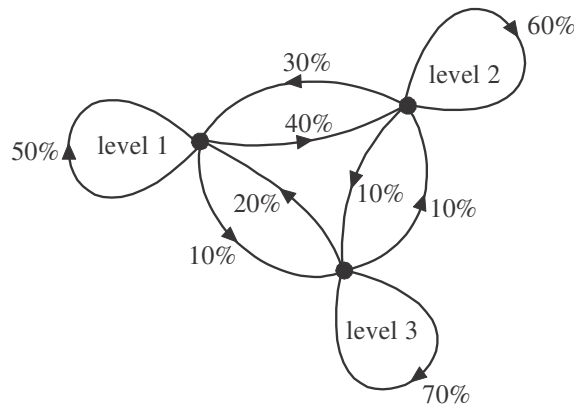
where G is a 1×3 matrix.
Write down matrix G .

1 mark

Question 2 (3 marks)

Staff at the shopping centre where this store is located who drive to work, can only park in allocated areas of a new car park. Staff are given weekly passes showing their option of parking on level 1, 2 or 3 of this new car park.

The transition diagram below shows the way in which staff are expected to change their choice of parking level from one week to the next.



- a. What percentage of staff who choose level 1 parking one week will choose level 3 parking the next week?

1 mark

- b. An incomplete transition matrix, T_1 , representing the transition diagram, is shown below.

$$T_1 = \begin{array}{ccc|l} & \text{this week} & & \\ & \text{level 1} & \text{level 2} & \text{level 3} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & - & 0.1 \\ - & 0.1 & 0.7 \end{bmatrix} & \text{level 1} & \text{level 2} & \text{level 3} \end{array} \text{ next week}$$

Complete the transition matrix T_1 by filling in the two missing entries.

1 mark

- c. The number of staff who parked on each of the levels in week 1 of the new car park being open, is given by the matrix N_1 , below.

$$N_1 = \begin{bmatrix} 65 \\ 91 \\ 82 \end{bmatrix} \begin{array}{l} \text{level 1} \\ \text{level 2} \\ \text{level 3} \end{array}$$

How many staff are expected to park on level 3 in week 2?

1 mark

Question 3 (5 marks)

Permanent staff at one of the department stores in the shopping centre must do a half-day training session every quarter.

They can choose one out of accounting (A), customer service (C), design (D) and health and safety (H) each quarter.

The way in which the staff are expected to change their choice of training session each quarter is shown in the transition matrix T_2 below.

$$T_2 = \begin{array}{c} \begin{array}{ccccc} & \text{this quarter} & & & \\ & A & C & D & H \\ \begin{bmatrix} 0.6 & 0.4 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.3 & 0.5 \end{bmatrix} & \begin{array}{l} A \\ C \\ D \\ H \end{array} & \text{next quarter} \end{array} \end{array}$$

The state matrix S_n gives the number of staff expected to choose each training session in the n^{th} quarter where

$$S_1 = \begin{bmatrix} 30 \\ 10 \\ 20 \\ 40 \end{bmatrix} \begin{array}{l} A \\ C \\ D \\ H \end{array}$$

The matrix rule $S_{n+1} = T_2 S_n$ can be used to find the state matrices for quarters 2, 3 and 4.

- a.** How many staff members are expected to choose the same training session for quarters 1 and 2? 1 mark

- b.** How many staff members are expected to choose health and safety (H) as their training session in quarter 2? 1 mark

- c.** Of the staff expected to choose accounting (*A*) in quarter 3, what percentage chose design (*D*) in quarter 2?

Round your answer to the nearest whole number.

2 marks

- d.** What is the least number of staff expected in one of the accounting (*A*) training sessions held over the first four quarters?

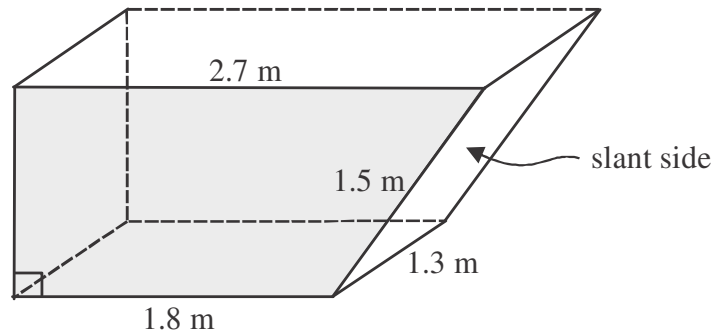
1 mark

Module 3 - Geometry and measurement

If you choose this module all questions must be answered.

Question 1 (3 marks)

A reception desk at an airport is in the shape of a trapezoidal prism. The front face of the desk is in the shape of a trapezium and is shaded in the diagram below.



For this desk, the length of the base is 1.8 m, the length of the top is 2.7 m, the length of the slant edge is 1.5 m and the depth is 1.3 m.

- a. What is the height of the desk, in metres? 1 mark

- b. Find the total surface area of the desk, including the top and the base, in square metres. 1 mark

- c. On the front face of the desk which is shaded, find the angle of elevation of the top right hand corner from the bottom right hand corner. Round your answer to the nearest whole degree. 1 mark

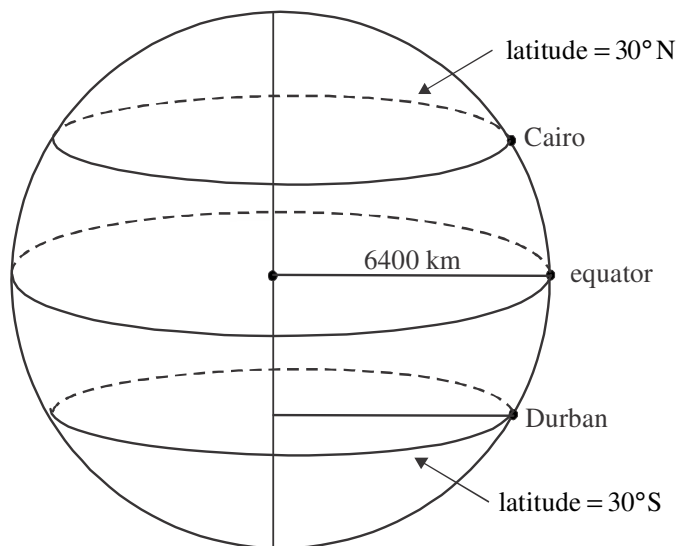
Question 2 (3 marks)

Kristy leaves Melbourne (38°S , 145°E) on Monday 6 March at 1.15pm.
 She travels for 22 hours and 30 minutes to reach Durban (30°S , 31°E) in South Africa.
 The time difference between Melbourne and Durban is 9 hours.

- a. What is the day and time that Kristy arrives in Durban? 1 mark

A few days later, Kristy travels from Durban (30°S , 31°E) to Cairo (30°N , 31°E) in Egypt.
 Both cities are shown on the diagram below as is the small circles of Earth at latitudes 30°S and 30°N .

Assume that the radius of Earth is 6 400 km.



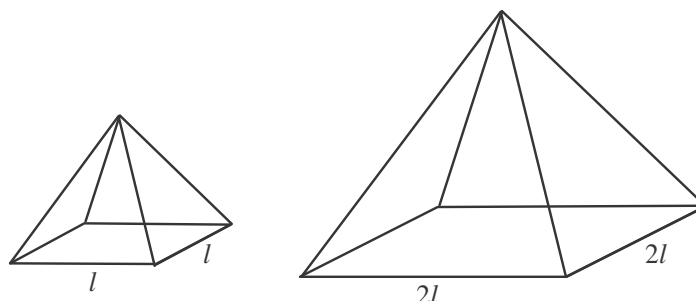
- b. What is the radius of the small circle of Earth at latitude 30°S ?
 Round your answer to the nearest kilometre. 1 mark

- c. Find the shortest great circle distance between Durban (30°S , 31°E) and Cairo (30°N , 31°E). Round your answer to the nearest kilometre. 1 mark

Question 3 (3 marks)

Whilst in Egypt, Kristy visits two pyramids.

- a. These pyramids, shown below, are similar in shape.



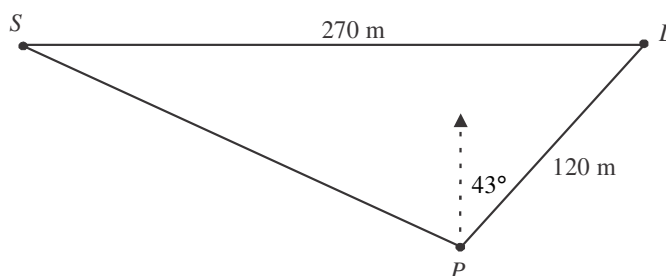
The sidelength of the square base of the smaller pyramid is l metres and for the larger pyramid it is double this, that is $2l$ metres.

The volume of the smaller pyramid is 200 000 cubic metres.

Find the volume, in cubic metres, of the larger pyramid.

1 mark

- b. To see the pyramids, Kristy walks for 120 m on a bearing of 043° from the carpark at P to the large pyramid at L . She then walks due west for 270 m to the small pyramid at S before walking in a straight line back to the carpark as shown in the diagram below.



- i. What is the bearing of the carpark at P from the large pyramid at L .

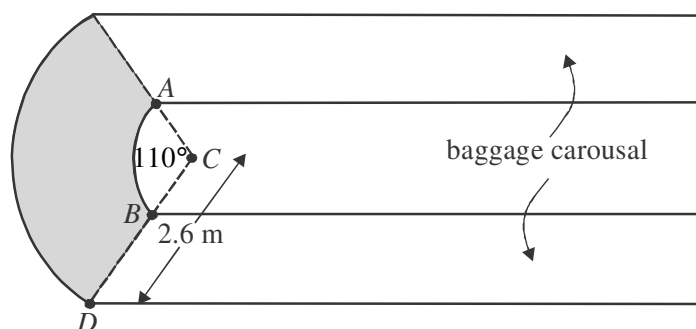
1 mark

- ii. Use the cosine rule to find the distance Kristy walks from the small pyramid at S to the carpark at P . Round your answer to the nearest metre.

1 mark

Question 4 (3 marks)

Kristy returns home and waits to collect her baggage at the baggage carousel. The shaded section of this baggage carousel shown below, is in the shape of a larger sector minus a smaller sector, with both sectors having an angle of 110° at their centre C . The larger sector has a radius CD of 2.6 metres.



The length of the arc AB of the smaller sector is 1.537 metres.

- a.** Show that the radius of the smaller sector is 0.8 metres, correct to one decimal place. 1 mark

- b.** What is the area of the shaded section of the baggage carousel? Round your answer to the nearest square metre. 2 marks

Further Mathematics formulas

Core - Data analysis

standardised score	$z = \frac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$

Module 1 - Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad \text{where } \det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \quad S_{n+1} = TS_n + B$

Module 2 - Networks and decision mathematics

Euler's formula	$v + f = e + 2$
-----------------	-----------------

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc \sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3} \times$ area of base $\times$ height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET

Mathematics Formula Sheets reproduced by permission; © VCAA 2017. The VCAA does not endorse or make any warranties regarding this study resource. Current and past VCAA VCE® exams and related content can be accessed directly at www.vcaa.vic.edu.au