



THE MATHEMATICAL
ASSOCIATION OF VICTORIA

2017 Meet the Assessors Mathematical Methods

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THE MATHEMATICAL
ASSOCIATION OF VICTORIA

Assessment Report for Exam 2



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MAV Solutions

- Fully worked solutions to 2016 Mathematical Methods VCAA paper are available from MAV shop
<https://shop.mav.vic.edu.au/index.php>



Statistics 2016 Total 80

- Highest score 80
- Lowest score 0
- n

(2015 15 634) (2014 15 557)



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Scaling 2016

- Mean 33.6 (33.8) and SD 8.4 (8.5)
- 20 (21)
- 25 (28)
- 30 (34)
- 35 (40)
- 40 (45)
- 45 (48) -1
- 50 (50) -1
- C:\Users\mcnamaraa\OneDrive - Trinity Grammar\MAV\scaling_report_16.pdf



Grade Distribution

- [vce_math_methods_cas_ga11.pdf](#)
- http://www.vcaa.vic.edu.au/vce/statistics/2011/section3/vce_math_methods_cas_ga11.pdf



Multiple Choice 2016

- 92% answered Question 1 and 90% answered Question 2 correctly.

1. A	2. B	3. C	4. D	5. D	6. A
7. C	8. B	9. D	10. D	11. D	12. D
13. E	14. E	15. C	16. E	17. A	18. B
19. E	20. E				



Question 1 A

92%

Question 1

The linear function $f: D \rightarrow R$, $f(x) = 5 - x$ has range $[-4, 5)$.

The domain D is

- A. $(0, 9]$
- B. $(0, 1]$
- C. $[5, -4)$
- D. $[-9, 0)$
- E. $[1, 9)$



Question 2

B

90%

Question 2

Let $f: R \rightarrow R$, $f(x) = 1 - 2 \cos\left(\frac{\pi x}{2}\right)$.

The period and range of this function are respectively

- A. 4 and $[-2, 2]$
- B. 4 and $[-1, 3]$
- C. 1 and $[-1, 3]$
- D. 4π and $[-1, 3]$
- E. 4π and $[-2, 2]$



D Question 10 (A 23)

52%

Question 10

For the curve $y = x^2 - 5$, the tangent to the curve will be parallel to the line connecting the positive x -intercept and the y -intercept when x is equal to

- A. $\sqrt{5}$
- B. 5
- C. -5
- D. $\frac{\sqrt{5}}{2}$
- E. $\frac{1}{\sqrt{5}}$

Coordinates of the x -intercept are $(\sqrt{5}, 0)$.

Coordinates of the y -intercept are $(0, -5)$.

Gradient of the line passing through these

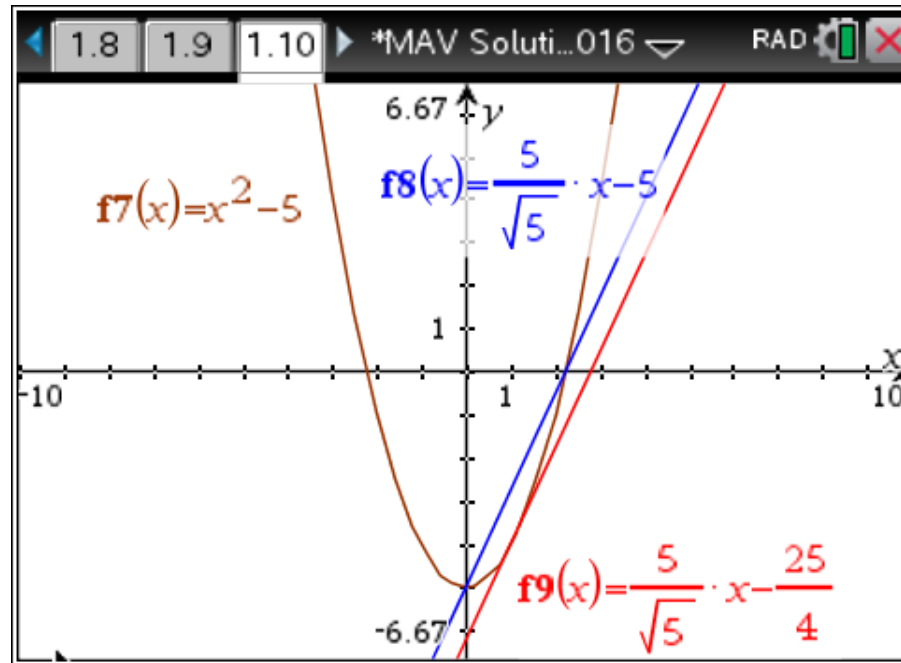
points is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-5)}{\sqrt{5} - 0} = \frac{5}{\sqrt{5}}$.

$$\frac{dy}{dx} = 2x = \frac{5}{\sqrt{5}}, x = \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$$



DQuestion 10 (A 23)

52%





D Question 12 (E 22)

52%

Question 12

The graph of a function f is obtained from the graph of the function g with rule $g(x) = \sqrt{2x-5}$ by a reflection in the x -axis followed by a dilation from the y -axis by a factor of $\frac{1}{2}$.

Which one of the following is the rule for the function f ?

A. $f(x) = \sqrt{5-4x}$

B. $f(x) = -\sqrt{x-5}$

C. $f(x) = \sqrt{x+5}$

D. $f(x) = -\sqrt{4x-5}$

E. $f(x) = -\sqrt{4x-10}$

$$g(x) = \sqrt{2x-5}$$

Reflection in the x -axis $g_1(x) = -\sqrt{2x-5}$

Dilation by a factor of $\frac{1}{2}$ from the y -axis

$$g_2(x) = -\sqrt{2\left(\frac{1}{2}x\right)-5} = -\sqrt{4x-5}$$



D Question 9 (E 21) 41%

Question 9

Given that $\frac{d(xe^{kx})}{dx} = (kx+1)e^{kx}$, then $\int xe^{kx} dx$ is equal to

A. $\frac{xe^{kx}}{kx+1} + c$

B. $\left(\frac{kx+1}{k}\right)e^{kx} + c$

C. $\frac{1}{k} \int e^{kx} dx$

D. $\frac{1}{k} \left(xe^{kx} - \int e^{kx} dx \right) + c$

E. $\frac{1}{k^2} (xe^{kx} - e^{kx}) + c$

$$\frac{d(xe^{kx})}{dx} = (kx+1)e^{kx}$$

$$\int ((kx+1)e^{kx}) dx = xe^{kx} + c_1$$

$$\int (kxe^{kx}) dx + \int (e^{kx}) dx = xe^{kx} + c_1$$

$$\int (kxe^{kx}) dx = xe^{kx} - \int (e^{kx}) dx + c_1$$

$$\int (xe^{kx}) dx = \frac{1}{k} (xe^{kx} - \int (e^{kx}) dx) + c$$



D Question 11 (C 20)

47%

Question 11

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y . Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x \log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

$$f(x) - f(y) = (y - x)f(xy)$$

$$f(x) = \frac{1}{x}$$

$$\text{LHS} = \frac{1}{x} - \frac{1}{y}$$

$$\text{RHS} = (y - x) \times \frac{1}{xy} = \frac{1}{x} - \frac{1}{y} = \text{LHS}$$



D Question 11 (C 20)

47%

1.10 1.11 1.12 ▶ *MAV Soluti...016 ▾ RAD  

Define $f(x) = \frac{1}{x}$ Done

$f(x) - f(y) = (y - x) \cdot f(x \cdot y)$ true

1.12 1.13 1.14 ▶ *MAV Soluti...016 ▾ RAD  

$f(x) - f(y) = (y - x) \cdot f(x \cdot y)$
 $x \cdot \ln(x) - y \cdot \ln(y) = x \cdot (x - y) \cdot \ln(x \cdot y) \cdot y$

Define $f(x) = \frac{1}{x^2}$ Done

$f(x) - f(y) = (y - x) \cdot f(x \cdot y)$ $\frac{1}{x^2} - \frac{1}{y^2} = \frac{-(x - y)}{x^2 \cdot y^2}$

Define $f(x) = x^2$ Done

$f(x) - f(y) = (y - x) \cdot f(x \cdot y)$
 $x^2 - y^2 = -x^2 \cdot (x - y) \cdot y^2$

Define $f(x) = x^2 + x^4$ Done

$f(x) - f(y) = (y - x) \cdot f(x \cdot y)$

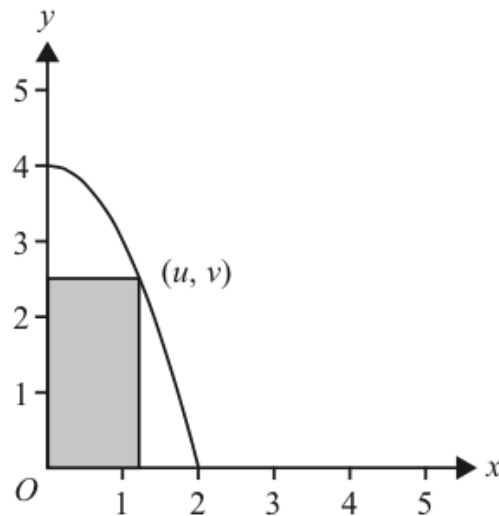


E Question 14 (B 21)

37%

Question 14

A rectangle is formed by using part of the coordinate axes and a point (u, v) , where $u > 0$ on the parabola $y = 4 - x^2$.





E Question 14 (B 21)

37%

Which one of the following is the maximum area of the rectangle?

A. 4

B. $\frac{2\sqrt{3}}{3}$

C. $\frac{8\sqrt{3} - 4}{3}$

D. $\frac{8}{3}$

E. $\frac{16\sqrt{3}}{9}$

Area of the rectangle = $x(4 - x^2)$

Let $A(x) = x(4 - x^2)$

Solve $\frac{dA}{dx} = -3x^2 + 4 = 0$, $x = \frac{2\sqrt{3}}{3}$

$A\left(\frac{2\sqrt{3}}{3}\right) = \frac{16\sqrt{3}}{9}$



E Question 19 (B 25)

15%

Question 19

Consider the discrete probability distribution with random variable X shown in the table below.

x	-1	0	b	$2b$	4
$\Pr(X = x)$	a	b	b	$2b$	0.2

The smallest and largest possible values of $E(X)$ are respectively

- A. -0.8 and 1
- B. -0.8 and 1.6
- C. 0 and 2.4
- D. 0.2125 and 1
- E. 0 and 1



E Question 19 (B 25)

15%

$$a + 4b + 0.2 = 1$$

$$a + 4b = 0.8 \dots (1)$$

$$E(X) = -a + 5b^2 + 0.8 \dots (2)$$

Substitute $0.8 - 4b$ for a into (2).

$E(X) = 5b^2 + 4b$, $0 \leq b \leq 0.2$, maximum value of b occurs when a is zero, $4b + 0.2 = 1$, $b = 0.2$

The smallest value for $E(X)$ is zero and the largest value one.

OR

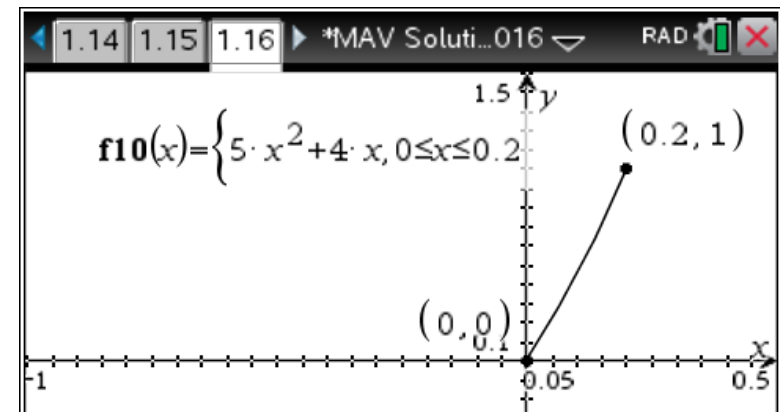
Trial and error

Let $a = 0$, $4b + 0.2 = 1$, $b = 0.2$

$$E(X) = -a + 5b^2 + 0.8 = 1$$

Let $b = 0$, $a = 0.8$

$$E(X) = -a + 5b^2 + 0.8 = 0$$





E Question 20 (C 30)

17%

Question 20

Consider the transformation T , defined as

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

The transformation T maps the graph of $y = f(x)$ onto the graph of $y = g(x)$.

If $\int_0^3 f(x) dx = 5$, then $\int_{-3}^0 g(x) dx$ is equal to

- A. 0
- B. 15
- C. 20
- D. 25
- E. 30



E Question 20 (C 30)

17%

Reflect in the y -axis $\int_0^3 f(x) dx = \int_{-3}^0 f(-x) dx = 5$

Dilate by a factor 3 from the x -axis

$$3 \int_{-3}^0 f(-x) dx = 15$$

Translate 5 units up.

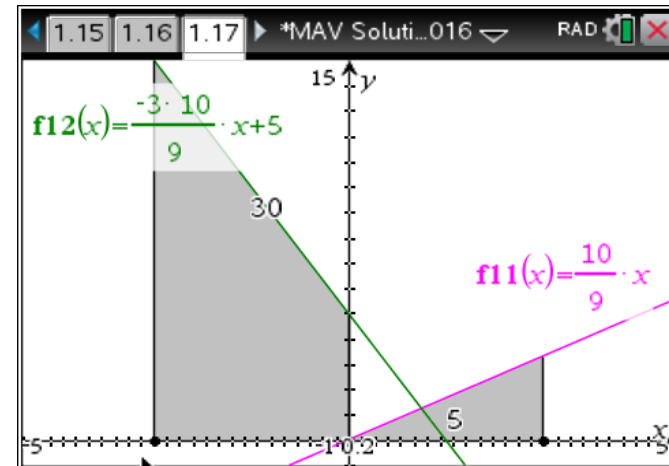
$$\int_{-3}^0 (3f(-x) + 5) dx = 15 + 15 = 30$$

OR

Make up an equation for f such as $f(x) = \frac{10}{9}x$

where $\int_0^3 f(x) dx = 5$. Hence $g(x) = -\frac{10}{3}x + 5$.

Calculator screen showing the integral calculation: $15 + \int_{-3}^0 5 dx = 30$





Extended Answer

- Give an exact answer unless otherwise stated.

~~$$\frac{1}{3} = 0.3$$~~

~~$$\frac{1}{3} \text{ OR } 0.3$$~~

- No calculator syntax.
- Show working for questions worth more than one mark. (Rule and answer)
- Work to more decimal places than the required answer.
- Use the variables that are given within the question. (SAC questions)



Extended Answer

- Reread questions.
- Take time when drawing graphs – scale axes, check if coordinates are required, one sharp line...
- Don't assume steps in *Show That* questions.
- Use brackets correctly.
- Transcribe formulas correctly (reread the calculator).



Extended Answer

- Put units in the final answer.
- Check that the final answer makes sense.
- Use the calculator.



Question 1a

Marks	0	1	2	Average
%	10	33	57	1.5

Question 1 (11 marks)

Let $f: [0, 8\pi] \rightarrow \mathbb{R}$, $f(x) = 2\cos\left(\frac{x}{2}\right) + \pi$.

a. Find the period and range of f .

$$\text{Period} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$$\text{Range} = [-2 + \pi, 2 + \pi]$$



Question 1b

Marks	0	1	Average
%	13	87	0.9

b. State the rule for the derivative function f' .

$$f'(x) = -\sin\left(\frac{x}{2}\right)$$

The screenshot shows a CASIO calculator interface with the following content:

- Top bar: 1.1, 1.2, 1.3, *MAVSoluti...wer, RAD, [X]
- Function definition: Define $f(x) = 2 \cdot \cos\left(\frac{x}{2}\right) + \pi | 0 \leq x \leq 8 \cdot \pi$ Done
- Derivative calculation: $y = \frac{d}{dx}(f(x))$
- Result: $y = \left\{ -\sin\left(\frac{x}{2}\right), 0 < x < 8 \cdot \pi \right\}$



Question 1c

Marks	0	1	Average
%	33	67	0.7

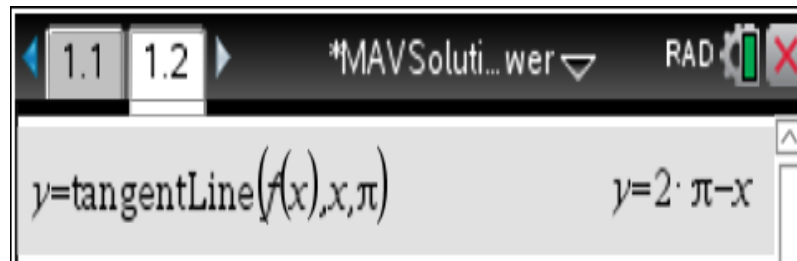
- c. Find the equation of the tangent to the graph of f at $x = \pi$.

$$f'(\pi) = -1, f(\pi) = \pi$$

Equation of the tangent line

$$y - \pi = -(x - \pi)$$

$$y = -x + 2\pi$$





Question 1d

Marks	0	1	2	Average
%	48	5	47	1.0

- d. Find the equations of the tangents to the graph of $f: [0, 8\pi] \rightarrow \mathbb{R}$, $f(x) = 2\cos\left(\frac{x}{2}\right) + \pi$ that have a gradient of 1.

$$f'(x) = 1, x = 3\pi \text{ or } x = 7\pi$$

For $x = 3\pi$

$$f(3\pi) = \pi$$

$$y - \pi = 1(x - 3\pi)$$

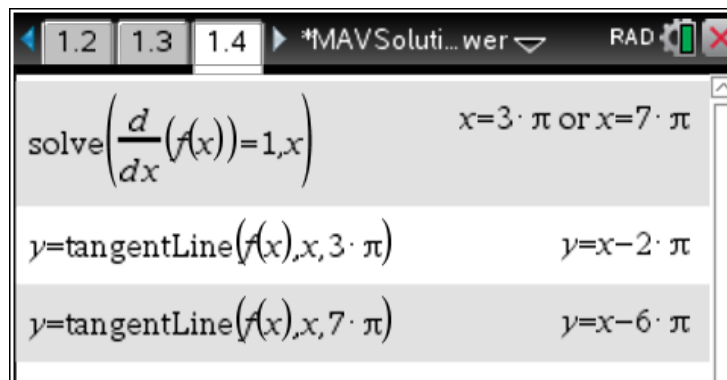
$$y = x - 2\pi$$

For $x = 7\pi$

$$f(7\pi) = \pi$$

$$y - \pi = 1(x - 7\pi)$$

$$y = x - 6\pi$$





Question 1e

Marks	0	1	2	3	Average
%	48	34	9	10	0.8

- e. The rule of f' can be obtained from the rule of f under a transformation T , such that

$$T: R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\pi \\ b \end{bmatrix}$$

Find the value of a and the value of b .

$$x' = x - \pi, x = x' + \pi$$

$$y' = ay + b, y = \frac{y' - b}{a}$$

Substitute the above equations in

$$y = 2 \cos\left(\frac{x}{2}\right) + \pi$$

$$\frac{y' - b}{a} = 2 \cos\left(\frac{x' + \pi}{2}\right) + \pi$$

$$y' = 2a \cos\left(\frac{x' + \pi}{2}\right) + a\pi + b$$

Using the identity $\cos\left(u + \frac{\pi}{2}\right) = -\sin(u)$

$$y' = -2a \sin\left(\frac{x'}{2}\right) + a\pi + b, \text{ therefore}$$

$$-2a = -1, a = \frac{1}{2}$$

$$a\pi + b = 0, b = -\frac{\pi}{2}$$



Question 1f

Marks	0	1	2	Average
%	49	4	47	1.0

f. Find the values of x , $0 \leq x \leq 8\pi$, such that $f(x) = 2f'(x) + \pi$.

Solve $f(x) = 2f'(x) + \pi$ for x .

$$2 \cos\left(\frac{x}{2}\right) + \pi = -2 \sin\left(\frac{x}{2}\right) + \pi$$

$$\tan\left(\frac{x}{2}\right) = -1, \quad \frac{x}{2} = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$$

$$x = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}$$

1.3 1.4 1.5 *MAVSoluti... wer RAD

$$\text{solve}\left(f(x) = -2 \cdot \sin\left(\frac{x}{2}\right) + \pi, x\right) | 0 < x < 8 \cdot \pi$$
$$x = \frac{3 \cdot \pi}{2} \text{ or } x = \frac{7 \cdot \pi}{2} \text{ or } x = \frac{11 \cdot \pi}{2} \text{ or } x = \frac{15 \cdot \pi}{2}$$



Question 2ai

Marks	0	1	Average
%	25	75	0.7

Question 2 (12 marks)

Consider the function $f(x) = -\frac{1}{3}(x+2)(x-1)^2$.

- a. i. Given that $g'(x) = f(x)$ and $g(0) = 1$, show that $g(x) = -\frac{x^4}{12} + \frac{x^2}{2} - \frac{2x}{3} + 1$.

$$f(x) = -\frac{1}{3}(x+2)(x-1)^2$$

$$g(x) = \int \left(-\frac{1}{3}(x+2)(x-1)^2 \right) dx$$

$$= -\frac{x^4}{12} + \frac{x^2}{2} - \frac{2x}{3} + c$$

$$g(0) = -\frac{0}{12} + \frac{0}{2} - \frac{0}{3} + c = 1, c = 1$$

$$g(x) = -\frac{x^4}{12} + \frac{x^2}{2} - \frac{2x}{3} + 1$$

The screenshot shows a graphing calculator window with the title bar "*MAVSoluti...wer" and a "RAD" indicator. The main display area shows the definition of the function $g(x) = \frac{-x^4}{12} + \frac{x^2}{2} - \frac{2 \cdot x}{3} + 1$. Below this, the integral is shown as $\int \left(\frac{-1}{3} \cdot (x+2) \cdot (x-1)^2 \right) dx + c$. The result of the integration is displayed at the bottom as $\frac{-x^4}{12} + \frac{x^2}{2} - \frac{2 \cdot x}{3} + c$. The interface includes navigation buttons (1.4, 1.5, 1.6) and a "Done" button.



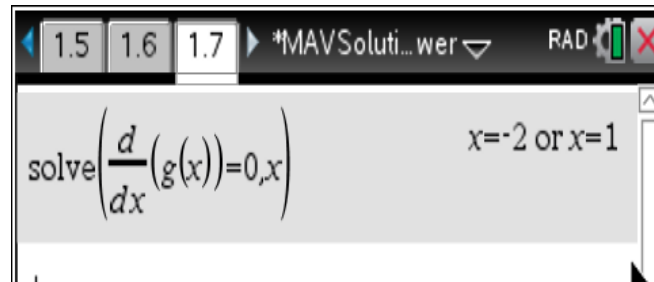
Question 2a ii

Marks	0	1	Average
%	18	82	0.8

ii. Find the values of x for which the graph of $y = g(x)$ has a stationary point.

Solve $g'(x) = 0$ for x .

$x = -2$ or $x = 1$

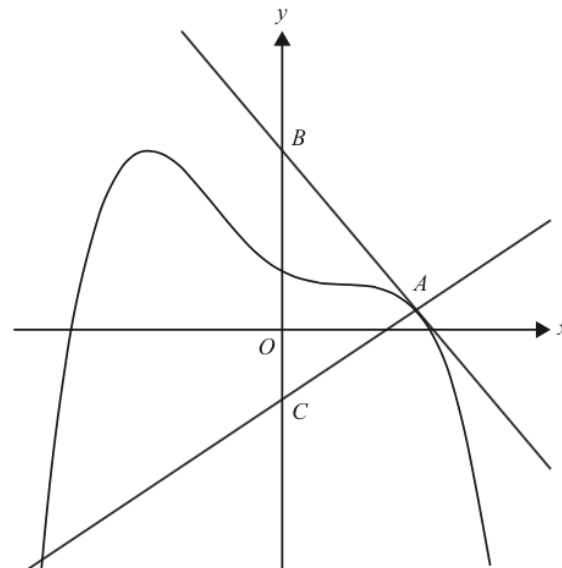




Question 2b

The diagram below shows part of the graph of $y = g(x)$, the tangent to the graph at $x = 2$ and a straight line drawn perpendicular to the tangent to the graph at $x = 2$. The equation of the tangent at the point A with coordinates $(2, g(2))$ is $y = 3 - \frac{4x}{3}$.

The tangent cuts the y -axis at B . The line perpendicular to the tangent cuts the y -axis at C .





Question 2bi

Marks	0	1	Average
%	15	85	0.9

- b.** **i.** Find the coordinates of B .

$$y = 3 - \frac{4x}{3}$$

$$x = 0, y = 3$$

The coordinates of B are $(0, 3)$.



Question 2bii

Marks	0	1	2	Average
%	27	18	55	1.3

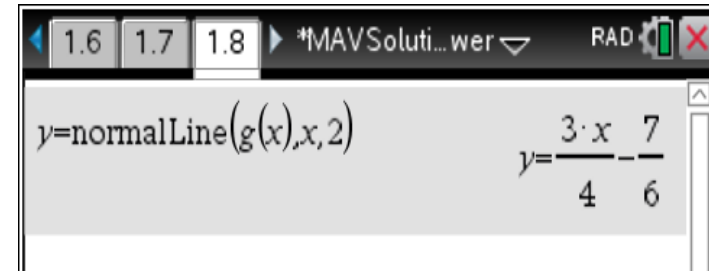
- ii. Find the equation of the line that passes through A and C and, hence, find the coordinates of C .

$$m = \frac{3}{4}, g(2) = \frac{1}{3}$$

$$y - \frac{1}{3} = \frac{3}{4}(x - 2)$$

$$y = \frac{3}{4}x - \frac{7}{6}$$

The coordinates of C are $\left(0, -\frac{7}{6}\right)$.





Question 2biii

Marks	0	1	2	Average
%	26	25	49	1.2

iii. Find the area of triangle ABC .

$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times \left(\frac{7}{6} + 3 \right) \times 2 \\ &= \frac{25}{6} \text{ units}^2\end{aligned}$$

OR

$$\text{Area} = \int_0^2 \left(\left(3 - \frac{4x}{3} \right) - \left(\frac{3x}{4} - \frac{7}{6} \right) \right) dx = \frac{25}{6}$$

1.7 1.8 1.9 *MAVSoluti...wer RAD X

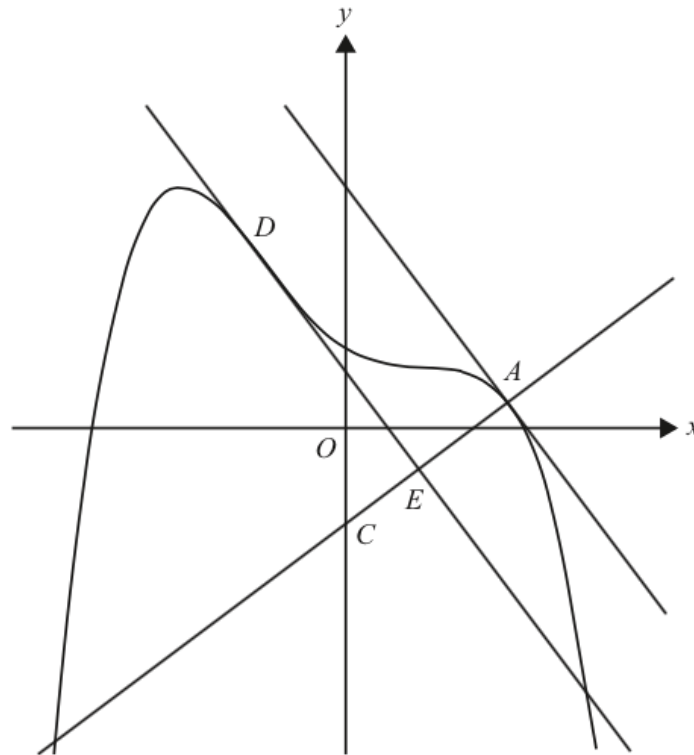
$\text{solve}\left(\frac{d}{dx}(g(x)) = -\frac{4}{3}, x\right)$ $x = -1 \text{ or } x = 2$

$g(-1)$ $\frac{25}{12}$



Question 2c

- c. The tangent at D is parallel to the tangent at A . It intersects the line passing through A and C at E .





Question 2ci

Marks	0	1	2	Average
%	53	6	40	0.9

i. Find the coordinates of D .

$$m = -\frac{4}{3}, \text{ solve } g'(x) = -\frac{4}{3} \text{ for } x$$

$$x = -1, g(-1) = \frac{25}{12}$$

The coordinates of D are $\left(-1, \frac{25}{12}\right)$.

The screenshot shows a TI-84 Plus calculator screen. The top status bar displays '1.7', '1.8', '1.9' in the window, 'MAVSoluti...wer' in the mode, and 'RAD' in the angle mode. The main display area shows the equation $\text{solve}\left(\frac{d}{dx}(g(x)) = -\frac{4}{3}, x\right)$ on the left and the solution $x = -1 \text{ or } x = 2$ on the right. Below this, the value $g(-1)$ is entered on the left, and the result $\frac{25}{12}$ is displayed on the right.



Question 2cii

ii. Find the length of AE .

$$A \left(2, \frac{1}{3} \right)$$

$$\text{Tangent at } x = -1, y = \frac{3}{4} - \frac{4}{3}x$$

$$\text{Solve } y = \frac{3}{4} - \frac{4}{3}x \text{ and } y = \frac{3}{4}x - \frac{7}{6} \text{ for } x \text{ and } y.$$

$$E \left(\frac{23}{25}, -\frac{143}{300} \right)$$

$$\text{Length} = \sqrt{\left(\frac{23}{25} - 2 \right)^2 + \left(-\frac{143}{300} - \frac{1}{3} \right)^2} = \frac{27}{20}$$

Marks	0	1	2	3	Average
%	62	6	5	27	1.0

1.8 1.9 1.10 *MAVSoluti... wer RAD

$y = \text{tangentLine}(g(x), x, -1)$

$y = \frac{3}{4} - \frac{4}{3}x$

$\text{solve}\left(y = \frac{3}{4} - \frac{4}{3}x \text{ and } y = \frac{3}{4}x - \frac{7}{6}, x, y\right)$

$x = \frac{23}{25} \text{ and } y = -\frac{143}{300}$

$\sqrt{\left(\frac{23}{25} - 2\right)^2 + \left(-\frac{143}{300} - \frac{1}{3}\right)^2}$

$\frac{27}{20}$



Question 3a

Marks	0	1	2	Average
%	16	8	76	1.6

Question 3 (16 marks)

A school has a class set of 22 new laptops kept in a recharging trolley. Provided each laptop is correctly plugged into the trolley after use, its battery recharges.

On a particular day, a class of 22 students uses the laptops. All laptop batteries are fully charged at the start of the lesson. Each student uses and returns exactly one laptop. The probability that a student does **not** correctly plug their laptop into the trolley at the end of the lesson is 10%. The correctness of any student's plugging-in is independent of any other student's correctness.

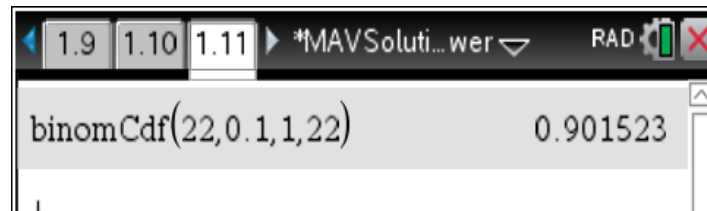
- a. Determine the probability that at least one of the laptops is **not** correctly plugged into the trolley at the end of the lesson. Give your answer correct to four decimal places.

Let X denote the number of laptops not correctly plugged in.

$$X \sim \text{Bi}(22, 0.1)$$

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) = 1 - 0.9^{22} = 0.9015$$

correct to four decimal places





Question 3b

Marks	0	1	2	Average
%	36	27	37	1.0

- b. A teacher observes that at least one of the returned laptops is not correctly plugged into the trolley.

Given this, find the probability that fewer than five laptops are **not** correctly plugged in. Give your answer correct to four decimal places.

$$\begin{aligned} & \Pr(X < 5 | X \geq 1) \\ &= \frac{\Pr(X \geq 1 \cap X < 5)}{\Pr(X \geq 1)} \\ &= \frac{\Pr(1 \leq X \leq 4)}{\Pr(X \geq 1)} \\ &= \frac{0.839389...}{0.901522...} \\ &= 0.9311 \text{ correct to four decimal places} \end{aligned}$$

binomCdf(22,0.1,1,22)	0.901523
binomCdf(22,0.1,1,4)	0.931079
	0.90152290978164



Question 3c

Marks	0	1	2	Average
%	29	23	48	1.2

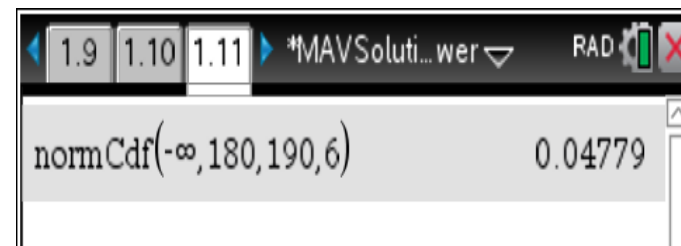
The time for which a laptop will work without recharging (the battery life) is normally distributed, with a mean of three hours and 10 minutes and standard deviation of six minutes. Suppose that the laptops remain out of the recharging trolley for three hours.

- c. For any one laptop, find the probability that it will stop working by the end of these three hours. Give your answer correct to four decimal places.

Let Y denote the battery life of a laptop.

$$Y \sim N(190, 6^2)$$

$\Pr(Y \leq 180) = 0.0478$ correct to four decimal places





Question 3d

Marks	0	1	2	3	Average
%	46	29	4	21	1.0

A supplier of laptops decides to take a sample of 100 new laptops from a number of different schools. For samples of size 100 from the population of laptops with a mean battery life of three hours and 10 minutes and standard deviation of six minutes, $\hat{P}$ is the random variable of the distribution of sample proportions of laptops with a battery life of less than three hours.

- d. Find the probability that $\Pr(\hat{P} \geq 0.06 \mid \hat{P} \geq 0.05)$. Give your answer correct to three decimal places. Do not use a normal approximation.

Let X_2 denote the number of laptops with battery life less than 3 hours.

$$X_2 \sim \text{Bi}(100, 0.04779\dots) \text{ and } \hat{P} = \frac{X_2}{n}$$

$$\begin{aligned} & \Pr(\hat{P} \geq 0.06 \mid \hat{P} \geq 0.05) \\ &= \frac{\Pr(\hat{P} \geq 0.05 \cap \hat{P} \geq 0.06)}{\Pr(\hat{P} \geq 0.05)} \\ &= \frac{\Pr(X_2 \geq 6)}{\Pr(X_2 \geq 5)} \\ &= \frac{0.34433\dots}{0.52340\dots} \\ &= 0.658 \end{aligned}$$

MAV Solutions

binomCdf(100,0.047790330359737,6,100)

binomCdf(100,0.047790330359737,5,100)

0.657879



Question 3e

Marks	0	1	2	Average
%	53	14	32	0.8

It is known that when laptops have been used regularly in a school for six months, their battery life is still normally distributed but the mean battery life drops to three hours. It is also known that only 12% of such laptops work for more than three hours and 10 minutes.

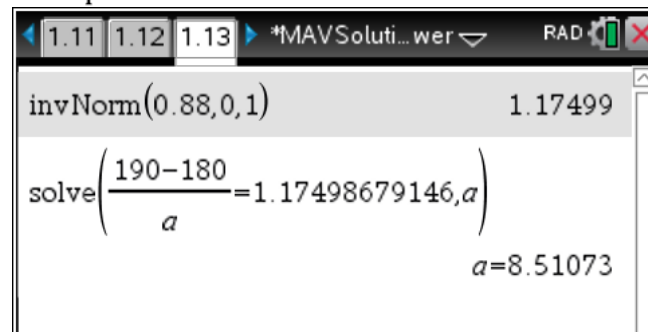
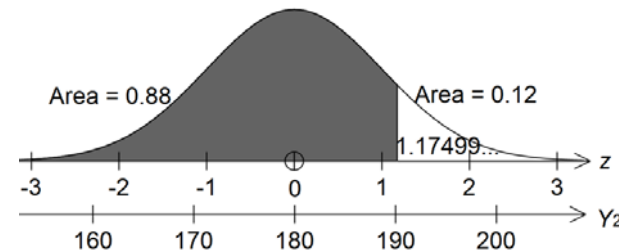
- e. Find the standard deviation for the normal distribution that applies to the battery life of laptops that have been used regularly in a school for six months, correct to four decimal places.

Let Y_2 denote battery life of laptops used for six months.

$$Y_2 \sim N(180, \sigma^2)$$

$$\frac{190-180}{\sigma} = 1.1749...$$

$\sigma = 8.5107$ correct to four decimal places





Question 3f

Marks	0	1	Average
%	79	21	0.2

The laptop supplier collects a sample of 100 laptops that have been used for six months from a number of different schools and tests their battery life. The laptop supplier wishes to estimate the proportion of such laptops with a battery life of less than three hours.

- f. Suppose the supplier tests the battery life of the laptops one at a time.

Find the probability that the first laptop found to have a battery life of less than three hours is the third one.

$$\Pr(Y_2 < 180) = \frac{1}{2}$$

$\Pr(\text{greater than 180, greater than 180, less$

$$\text{than 180}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$



Question 3g

Marks	0	1	Average
%	59	41	0.4

The laptop supplier finds that, in a particular sample of 100 laptops, six of them have a battery life of less than three hours.

- g. Determine the 95% confidence interval for the supplier's estimate of the proportion of interest. Give values correct to two decimal places.

$$n = 100, \quad \hat{p} = \frac{6}{100} = 0.06$$

$$CI \approx 0.06 \pm 1.96 \sqrt{\frac{0.06 \times (1 - 0.06)}{100}}$$

$$CI \approx (0.01345..., 0.10654...)$$

(0.01, 0.11) correct to two decimal places

"Title"	"1-Prop z Interval"
"CLower"	0.013453
"CUpper"	0.106547
"p"	0.06
"ME"	0.046547
"n"	100.



Question 3hi

Marks	0	1	Average
%	33	67	0.7

- h. The supplier also provides laptops to businesses. The probability density function for battery life, x (in minutes), of a laptop after six months of use in a business is

$$f(x) = \begin{cases} \frac{(210-x)e^{\frac{x-210}{20}}}{400} & 0 \leq x \leq 210 \\ 0 & \text{elsewhere} \end{cases}$$

- i. Find the **mean** battery life, in minutes, of a laptop with six months of business use, correct to two decimal places.

$$E(X) = \int_0^{210} (x \times f(x)) dx = 170.01 \text{ correct to two decimal places}$$

The screenshot shows a CAS calculator interface with the following content:

- Top bar: 1.13, 1.14, 1.15, *MAV Soluti...wer, RAD, and a close button.
- Define $f(x) = \begin{cases} \frac{(210-x) \cdot e^{\frac{x-210}{20}}}{400}, & 0 \leq x \leq 210 \\ 0, & \text{Else} \end{cases}$
- Below the definition, the word "Done" is visible.
- Below the definition, the integral $\int_0^{210} (x \cdot f(x)) dx$ is shown, with the result 170.007 displayed to the right.



Question 3hii

Marks	0	1	2	Average
%	27	15	58	1.3

- ii. Find the **median** battery life, in minutes, of a laptop with six months of business use, correct to two decimal places.

Solve $\int_0^m (f(x))dx = \frac{1}{2}$ for m .

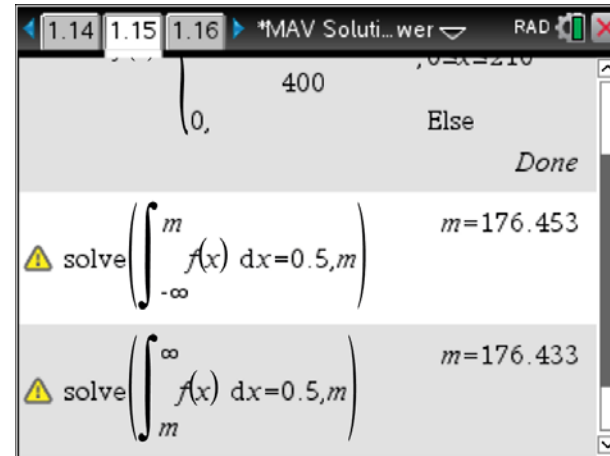
$m = 176.45$ correct to two decimal places

OR

Solve $\int_m^{210} (f(x))dx = \frac{1}{2}$ for m .

$m = 176.43$ correct to two decimal places

The answers differ slightly as the area under the curve is not exactly 1.





Question 4a

Marks	0	1	2	Average
%	42	6	52	1.1

Question 4 (21 marks)

- a. Express $\frac{2x+1}{x+2}$ in the form $a + \frac{b}{x+2}$, where a and b are non-zero integers.

$$\begin{array}{r} x+2 \overline{) 2x+1} \\ \underline{2x+4} \\ -3 \end{array}$$
$$\frac{2x+1}{x+2} = 2 + \frac{-3}{x+2}$$

Alternatively

$$\begin{aligned} \frac{2x+1}{x+2} &= \frac{2(x+2)-3}{x+2} = \frac{\cancel{2(x+2)}-3}{\cancel{x+2}} = \frac{3}{x+2} \\ &= 2 + \frac{-3}{x+2} \end{aligned}$$

The screenshot shows a CAS calculator window with the following content:

- Top bar: 1.16 1.17 1.18 *MAVSoluti...wer RAD
- Input: Define $f(x) = \frac{2 \cdot x + 1}{x + 2}$ Done
- Input: $\text{propFrac}(f(x))$
- Output: $2 - \frac{3}{x + 2}$



Question 4bi

Marks	0	1	2	Average
%	19	26	56	1.4

b. Let $f: R \setminus \{-2\} \rightarrow R$, $f(x) = \frac{2x+1}{x+2}$.

i. Find the rule and domain of f^{-1} , the inverse function of f .

$$f(x) = \frac{2x+1}{x+2}$$

$$\text{Let } y = \frac{2x+1}{x+2}$$

Inverse swap x and y

$$x = \frac{2y+1}{y+2}$$

$$y = \frac{-3}{x-2} - 2$$

Range of f is $R \setminus \{2\}$

Domain of f^{-1} is $R \setminus \{2\}$

$$f^{-1}: R \setminus \{2\} \rightarrow R, f^{-1}(x) = \frac{-3}{x-2} - 2$$

The screenshot shows a CASIO calculator interface with the following steps:

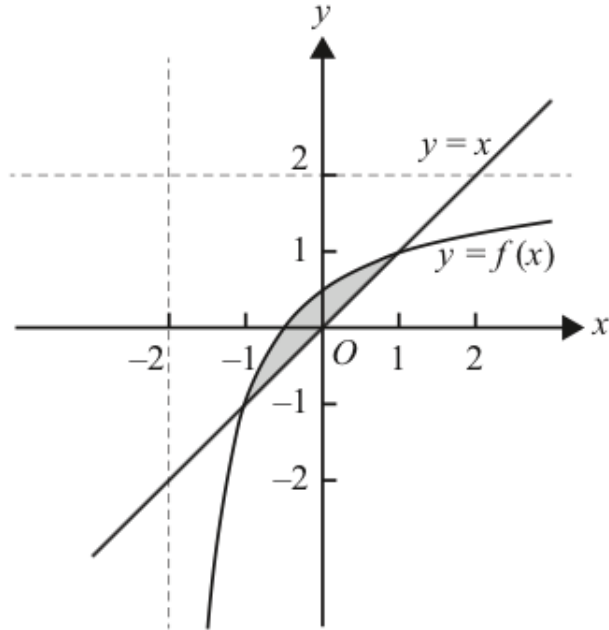
- Input: $\text{solve}(f(y)=x, y)$
- Output: $y = \frac{-(2 \cdot x - 1)}{x - 2}$
- Input: $\text{propFrac}\left(\frac{-(2 \cdot x - 1)}{x - 2}\right)$
- Output: $\frac{-3}{x - 2} - 2$



Question 4bii

Marks	0	1	Average
%	38	62	0.6

ii. Part of the graphs of f and $y = x$ are shown in the diagram below.



Solve $f(x) = x$, $x = -1$ or $x = 1$

$$\begin{aligned}\text{Area} &= \int_{-1}^1 (f(x) - x) dx \\ &= -3\log_e(3) + 4 \text{ units}^2\end{aligned}$$

1.19 1.20 1.21 *MAVSoluti...wer RAD

$\text{solve}(f(x)=x,x)$ $x=-1 \text{ or } x=1$

$\int_{-1}^1 (f(x)-x)dx$ $4-3 \cdot \ln(3)$

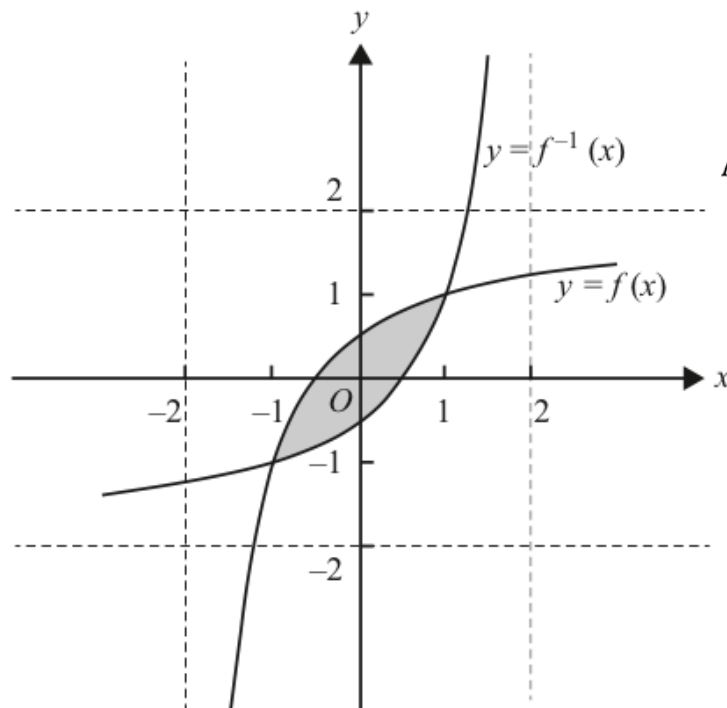
Find the area of the shaded region.



Question 4biii

Marks	0	1	Average
%	38	62	0.6

iii. Part of the graphs of f and f^{-1} are shown in the diagram below.



$$\begin{aligned}\text{Area} &= \int_{-1}^1 (f(x) - f^{-1}(x)) dx \\ &= -6 \log_e(3) + 8 \text{ (double bii)}\end{aligned}$$

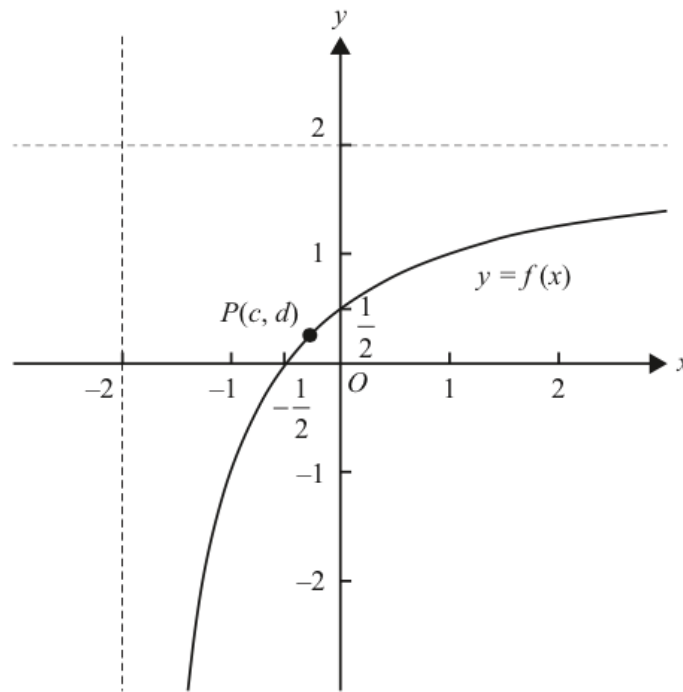
Find the area of the shaded region.



Question 4c

Marks	0	1	2	3	Average
%	69	4	14	14	0.7

Part of the graph of f is shown in the diagram below.



$$P\left(x, \frac{2x+1}{x+2}\right)$$

$$\text{distance } y = \sqrt{x^2 + \left(\frac{2x+1}{x+2}\right)^2}, \quad -2 < x < \infty$$

$$\frac{dy}{dx} = 0$$

$$c = \sqrt{3} - 2, \quad d = -\sqrt{3} + 2$$

$$\text{distance} = 2\sqrt{2} - \sqrt{6}$$

The point $P(c, d)$ is on the graph of f .

Find the exact values of c and d such that the distance of this point to the origin is a minimum, and find this minimum distance.



Question 4c

Marks	0	1	2	3	Average
%	69	4	14	14	0.7

The screenshot shows a CAS calculator window with the following content:

1.19 1.20 1.21 *MAVSoluti... wer RAD

solve $\left(\frac{d}{dx} \left(\sqrt{(f(x))^2 + x^2} \right) = 0, x \right)$

$x = -(\sqrt{3} + 2)$ or $x = \sqrt{3} - 2$

$\sqrt{(f(x))^2 + x^2} |_{x=\sqrt{3}-2} \quad 2 \cdot \sqrt{2} - \sqrt{6}$

N.B. the solution $x = -(\sqrt{3} + 2)$ is outside the domain $x \in (-2, \infty)$.



Question 4d

Marks	0	1	2	Average
%	90	4	6	0.2

Let $g: (-k, \infty) \rightarrow \mathbb{R}$, $g(x) = \frac{kx+1}{x+k}$, where $k > 1$.

d. Show that $x_1 < x_2$ implies that $g(x_1) < g(x_2)$, where $x_1 \in (-k, \infty)$ and $x_2 \in (-k, \infty)$

$$g'(x) = \frac{k^2 - 1}{(k+x)^2} > 0 \text{ for } x \in \mathbb{R}, k > 1$$

Hence $g(x)$ is a strictly increasing function.

OR

Let $x_2 > x_1 > -k$, $k > 1$

$$g(x_2) - g(x_1) = \frac{(x_2 - x_1)(k^2 - 1)}{(x_1 + k)(x_2 + k)}$$

$$x_2 - x_1 > 0, k^2 - 1 > 0$$

If $x_2 > x_1 > -k$, then $-x_2 < -x_1 < k$

Hence $k + x_1 > 0$ and $k + x_2 > 0$

Hence $g(x_2) - g(x_1) > 0$.

So $g(x)$ is a strictly increasing function.

OR

$$g(x_1) = k - \frac{k^2 - 1}{x_1 + k}, g(x_2) = k - \frac{k^2 - 1}{x_2 + k}$$

$$\text{If } x_1 < x_2, \frac{k^2 - 1}{x_1 + k} > \frac{k^2 - 1}{x_2 + k}, k^2 - 1 > 0, k > 1$$

$$\text{Hence, } k - \frac{k^2 - 1}{x_1 + k} < k - \frac{k^2 - 1}{x_2 + k}$$

$$g(x_1) < g(x_2)$$



Question 4ei

Marks	0	1	2	Average
%	60	16	24	0.6

- e. i. Let X be the point of intersection of the graphs of $y = g(x)$ and $y = -x$.

Find the coordinates of X in terms of k .

Solve $g(x) = -x$ for x .

$$x = -k + \sqrt{k^2 - 1}$$

$$y = k - \sqrt{k^2 - 1}$$

$$X(-k + \sqrt{k^2 - 1}, k - \sqrt{k^2 - 1})$$

The screenshot shows a CASIO calculator interface with the following steps:

- Define $g(x) = \frac{k \cdot x + 1}{x + k}$ (labeled "Done")
- Solve $(g(x) = -x, x)$
- Result: $x = -(\sqrt{k^2 - 1} + k)$ or $x = \sqrt{k^2 - 1} - k$

N.B. the solution $x = -k - \sqrt{k^2 - 1}$ is outside the domain $x \in (-k, \infty)$.



Question 4eii

Marks	0	1	2	Average
%	51	11	38	0.9

- ii. Find the value of k for which the coordinates of X are $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

Solve $-k + \sqrt{k^2 + 1} = -\frac{1}{2}$ for k .

$$k = \frac{5}{4}$$

Alternatively

Solve $g\left(-\frac{1}{2}\right) = \frac{1}{2}$ for k .

$$k = \frac{5}{4}$$

The screenshot shows a TI-Nspire calculator interface. The top status bar indicates the current page is 1.22, the mode is RAD, and there is a document icon. The main display area shows the following steps:

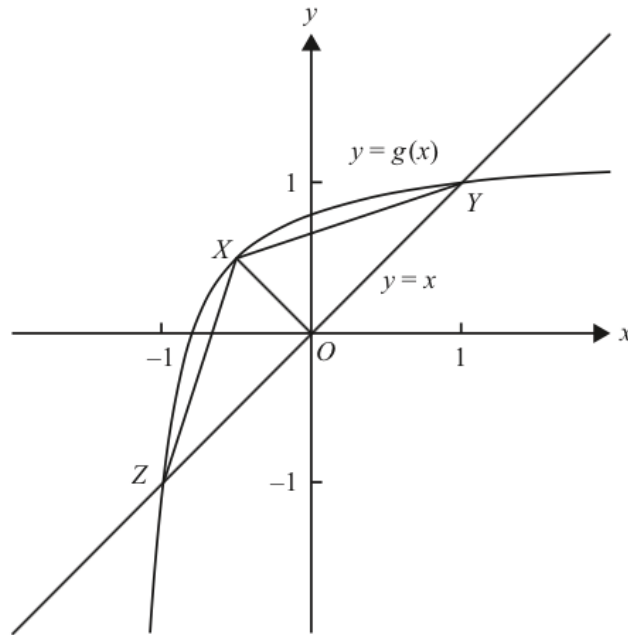
- Define $g(x) = \frac{k \cdot x + 1}{x + k}$ (The word "Done" is visible to the right of the expression.)
- solve($g(x) = -x, x$)
- The result of the solve command is displayed as $x = -(\sqrt{k^2 - 1} + k)$ or $x = \sqrt{k^2 - 1} - k$.
- The final step is solve($g\left(\frac{-1}{2}\right) = \frac{1}{2}, k$), which results in $k = \frac{5}{4}$.



Question 4eiii

Marks	0	1	2	Average
%	91	5	3	0.1

- iii. Let $Z(-1, -1)$, $Y(1, 1)$ and X be the vertices of the triangle XYZ . Let $s(k)$ be the square of the area of triangle XYZ .



Find the values of k such that $s(k) \geq 1$.

$$YZ = 2\sqrt{2}$$

$$s(k) = \left(\frac{1}{2} \times 2\sqrt{2} \times \left(\sqrt{(-k + \sqrt{k^2 + 1})^2 + (k - \sqrt{k^2 + 1})^2} \right) \right)^2$$

$$= 4(\sqrt{k^2 - 1} - k)^2$$

Solve $s(k) > 1$ for k , $1 < k \leq \frac{5}{4}$

$$\left(\frac{1}{2} \cdot 2 \cdot \sqrt{2} \cdot \sqrt{(-k + \sqrt{k^2 - 1})^2 + (k - \sqrt{k^2 - 1})^2} \right)^2$$

$$4 \cdot (\sqrt{k^2 - 1} - k)^2$$

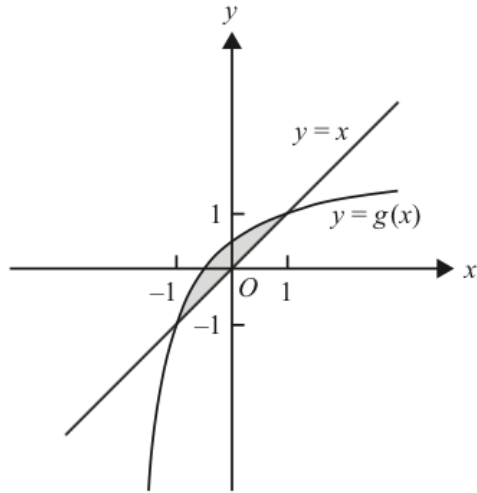
$$\text{solve} \left(4 \cdot (\sqrt{k^2 - 1} - k)^2 > 1, k \right) \quad 1 \leq k < \frac{5}{4} \text{ or } k \leq -1$$



Question 4fi

Marks	0	1	2	Average
%	61	20	18	0.6

- f. The graph of g and the line $y = x$ enclose a region of the plane. The region is shown shaded in the diagram below.



$$\begin{aligned} A(k) &= \int_{-1}^1 (g(x) - x) dx \\ &= (k^2 - 1) \log_e \left(\frac{k-1}{k+1} \right) + 2k \end{aligned}$$

Let $A(k)$ be the rule of the function A that gives the area of this enclosed region. The domain of A is $(1, \infty)$.

- i. Give the rule for $A(k)$.



Question 4fii

Marks	0	1	2	Average
%	94	4	2	0.1

ii. Show that $0 < A(k) < 2$ for all $k > 1$.

$g(x)$ is strictly increasing over the interval $[-1, 1]$ and $g(x) > x$, $k > 1$, then $A(k) > 0$ and will always be in the region bounded by the triangle formed by the lines $x = -1$, $y = x$ and $y = 1$. The area of this triangle is 2 units². Hence $0 < A(k) < 2$.

OR

$$\lim_{k \rightarrow \infty} (A(k)) = 0$$

$$\lim_{k \rightarrow 1} (A(k)) = 2$$

